

Defining causal effects

UCLA SOCIOL 114: Social Data Science
Winter 2025

28 Jan 2025

Learning goals for today

By the end of class, you will be able to

- ▶ explain the fundamental problem of causal inference and the need for causal arguments
- ▶ define potential outcomes
- ▶ recall mathematical concepts from probability
 - ▶ random variables
 - ▶ expectation
 - ▶ conditional expectation

Causal claims hinge on arguments, not on data



Left photo: By Fernando Frazão/Agência Brasil - http://agenciabrasil.ebc.com.br/sites/_agenciabrasil2013/files/fotos/1035034-_mg_0802_04.08.16.jpg, CC BY 3.0, <https://commons.wikimedia.org/w/index.php?curid=50548410>
Right photo: By Agencia Brasil Fotografias - EUA levam ouro na ginástica artística feminina; Brasil fica em 8 lugar, CC BY 2.0, <https://commons.wikimedia.org/w/index.php?curid=50584648>

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- ▶ Swinging on the uneven bars causes a person to win a gold medal.

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	Swing	Do not swing	of swinging
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Ian	?	No (0)	?

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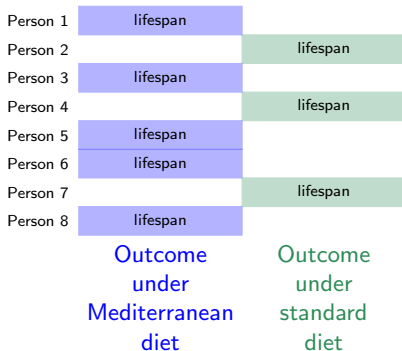
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Fundamental problem of causal inference

Holland 1986

Descriptive evidence



Fundamental problem of causal inference

Holland 1986

Descriptive evidence



Causal claim



Person 1	lifespan	
Person 2		lifespan
Person 3	lifespan	
Person 4		lifespan
Person 5	lifespan	
Person 6	lifespan	
Person 7		lifespan
Person 8	lifespan	

Outcome
under
Mediterranean
diet

Outcome
under
standard
diet

lifespan	lifespan
lifespan	lifespan
lifespan	lifespan
lifespan	lifespan
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Fundamental problem of causal inference

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Causal claim



Person 1	lifespan	missing
Person 2	missing	lifespan
Person 3	lifespan	missing
Person 4	missing	lifespan
Person 5	lifespan	missing
Person 6	lifespan	missing
Person 7	missing	lifespan
Person 8	lifespan	missing

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Causal claim



Causal inference is a **missing data** problem

Person 1	lifespan	missing
Person 2	missing	lifespan
Person 3	lifespan	missing
Person 4	missing	lifespan
Person 5	lifespan	missing
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Mathematical notation: Potential outcomes

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Y_i Outcome

Whether person i survived

Mathematical notation: Potential outcomes

Y_i	Outcome	Whether person i survived
A_i	Treatment	Whether person i ate a Mediterranean diet

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Examples:

$Y_{\text{Ian}} = \text{survived}$

Ian survived

Mathematical notation: Potential outcomes

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Examples:

$Y_{\text{Ian}} = \text{survived}$	Ian survived
$A_{\text{Ian}} = \text{MediterraneanDiet}$	Ian ate a Mediterranean diet

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Examples:

$Y_{\text{Ian}} = \text{survived}$	Ian survived
$A_{\text{Ian}} = \text{MediterraneanDiet}$	Ian ate a Mediterranean diet
$Y_{\text{Ian}}^{\text{MediterraneanDiet}} = \text{survived}$	Ian would survive on a Mediterranean diet

Mathematical notation: Potential outcomes

Y_i	Outcome	Whether person i survived
A_i	Treatment	Whether person i ate a Mediterranean diet
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Examples:

Y_{Ian}	= survived	Ian survived
A_{Ian}	= MediterraneanDiet	Ian ate a Mediterranean diet
$Y_{\text{Ian}}^{\text{MediterraneanDiet}}$	= survived	Ian would survive on a Mediterranean diet
$Y_{\text{Ian}}^{\text{StandardDiet}}$	= died	Ian would die on a standard diet

Mathematical notation: Potential outcomes

Y_i	Outcome	Whether person i survived
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Examples:

Y_{Ian}	= survived	Ian survived
A_{Ian}	= MediterraneanDiet	Ian ate a Mediterranean diet
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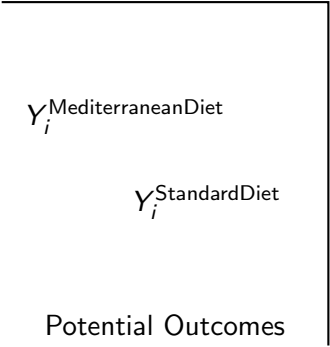
Discuss.

Which potential outcome is observed?

Which is counterfactual?

The consistency assumption

The consistency assumption

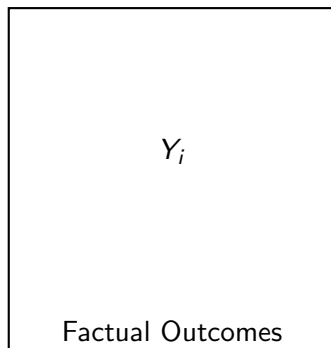
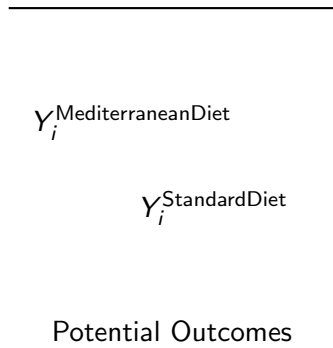


$Y_i^{\text{MediterraneanDiet}}$

$Y_i^{\text{StandardDiet}}$

Potential Outcomes

The consistency assumption



The consistency assumption

Consistency Assumption

$$Y_i^{A_i} = Y_i$$

$Y_i^{\text{MediterraneanDiet}}$

$Y_i^{\text{StandardDiet}}$

Potential Outcomes

Y_i

Factual Outcomes

Mathematical notation: Potential outcomes are fixed

A person's potential outcome is a **fixed quantity**

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The outcome for a random person is a **random variable**

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The outcome for a random person is a **random variable**

- ▶ Draw a random person from the population

Mathematical notation: Potential outcomes are fixed

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The outcome for a random person is a **random variable**

- ▶ Draw a random person from the population
- ▶ Assign them a Mediterranean diet

Mathematical notation: Potential outcomes are fixed

A person's potential outcome is a **fixed quantity**

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The outcome for a random person is a **random variable**

- ▶ Draw a random person from the population
- ▶ Assign them a Mediterranean diet
- ▶ The outcome $Y^{\text{MediterraneanDiet}}$ is a random variable:
 - ▶ takes the value `survived` if we randomly sample some people
 - ▶ takes the value `died` if we randomly sample others

Mathematical notation: Potential outcomes are fixed

A person's potential outcome is a **fixed quantity**

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Check for understanding:

Does it make sense to write $V(Y_i^a)$? How about $V(Y^a)$

Notation: Expectation operator

The **expectation operator** $E()$ denotes the population mean

$$E(Y^a) = \frac{1}{n} \sum_{i=1}^n Y_i^a$$

The quantity Y^a inside the expectation must be a random variable

Notation: Expectation operator

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A **conditional expectation** is denoted with a vertical bar

$$E(Y \mid A = a) = \frac{1}{n_a} \sum_{i:A_i=a} Y_i$$

Practice: How would you say this in English?

We might wonder how a person's earnings relate to whether they hold a college degree

1. $E(\text{Earnings} \mid \text{Degree} = \text{TRUE}) > E(\text{Earnings} \mid \text{Degree} = \text{FALSE})$

2. $E(\text{Earnings}^{\text{Degree}=\text{TRUE}}) > E(\text{Earnings}^{\text{Degree}=\text{FALSE}})$

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► Average earnings are higher among those with college degrees

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► On average, a degree causes higher earnings

Practice: How would you write this in math?

1. On average, students who do the homework learn more than those who don't
2. On average, doing the homework causes more learning

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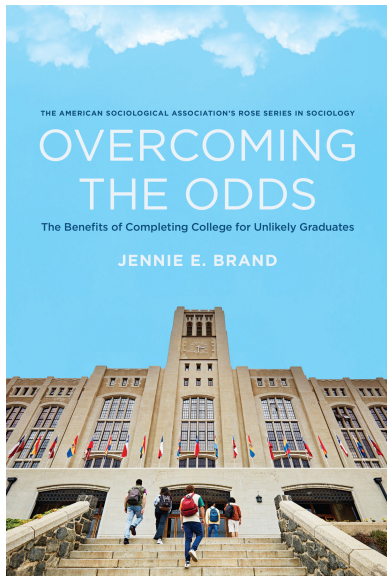
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An example about inequality

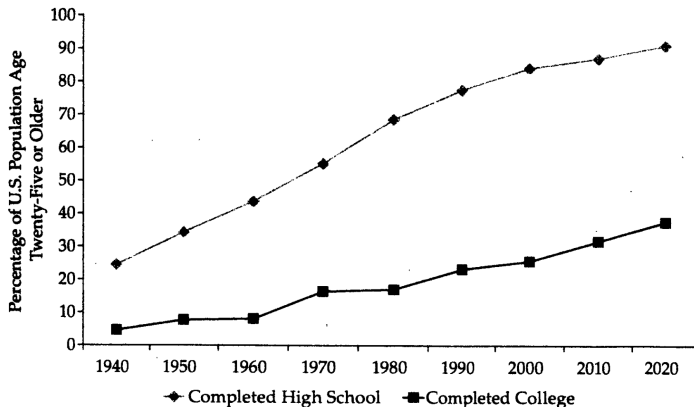


Americans' education in 1900

(Brand 2023 p. 6)

- ▶ 6% graduated from high school
- ▶ 3% graduated from college

Figure 1.1 High School and Four-Year College Completion Rates, 1940–2020



Source: U.S. Census Bureau, March Current Population Survey and Annual Social and Economic Supplement to the Current Population Survey.

(Brand 2023)

Why did education expand?

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- ▶ Public investment in college
 - ▶ Morrill Act (1862) sold land to establish colleges
 - ▶ G.I. Bill (1944) funded veterans' college

Why did education expand?

- ▶ Public investment in college
 - ▶ Morrill Act (1862) sold land to establish colleges
 - ▶ G.I. Bill (1944) funded veterans' college
- ▶ Rising labor market demand for skills

We would like to know whether **college pays off**:
does it have positive effects on desired outcomes?

Mathematical notation for two types of claims

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People with
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earn more

A college degree
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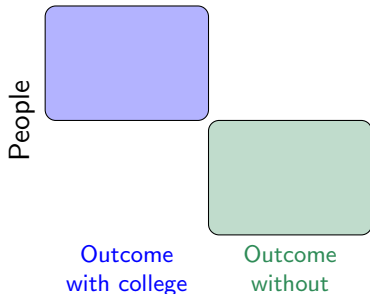
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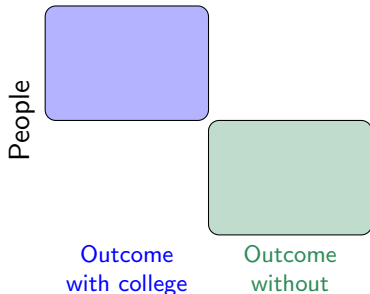
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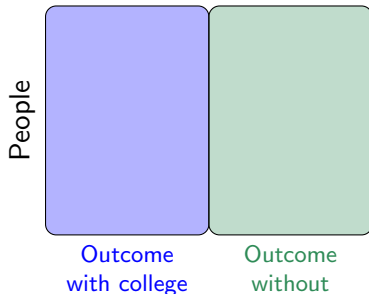
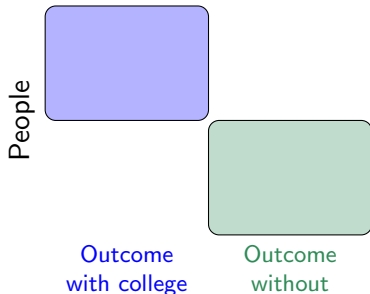
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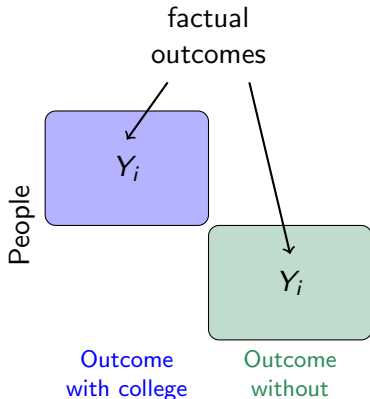
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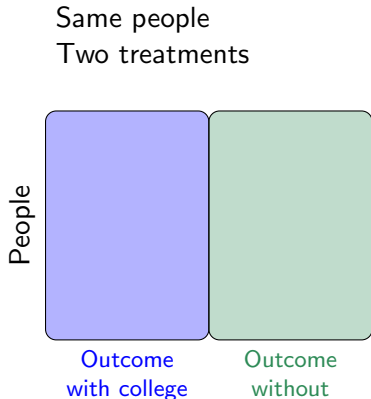


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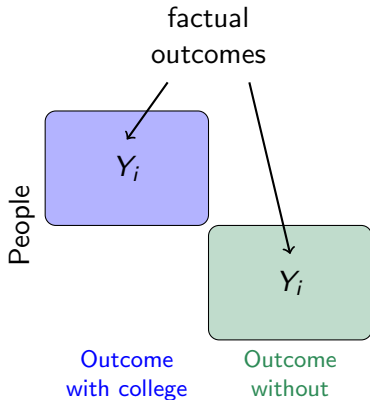


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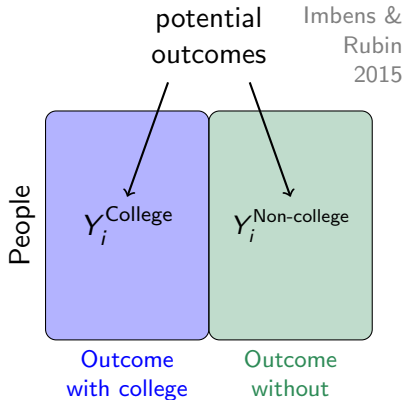


Mathematical notation for two types of claims

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The fundamental problem of causal inference

The data

Each Row is a Person

Y^{College} Nick	
Y^{College} William	
	$Y^{\text{Non-college}}$ Rich
Y^{College} Diego	
	$Y^{\text{Non-college}}$ Javier
	$Y^{\text{Non-college}}$ Jesús

Outcome under treatment Outcome under control

Holland 1986

The fundamental problem of causal inference

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The data	
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Y^{College} William	
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Outcome under treatment	Outcome under control

The claim	
Y^{College} Nick	\longleftrightarrow $Y^{\text{Non-college}}$ Nick
Y^{College} William	\longleftrightarrow $Y^{\text{Non-college}}$ William
Y^{College} Rich	\longleftrightarrow $Y^{\text{Non-college}}$ Rich
Y^{College} Diego	\longleftrightarrow $Y^{\text{Non-college}}$ Diego
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The fundamental problem of causal inference

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Each Row is a Person	Y^{College} Nick	?
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	Y^{College} Diego	?
	?	$Y^{\text{Non-college}}$ Javier
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Counterfactuals are **not** observed

The claim

Y^{College} Nick	\longleftrightarrow	$Y^{\text{Non-college}}$ Nick
Y^{College} William	\longleftrightarrow	$Y^{\text{Non-college}}$ William
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Holland 1986

Preview: Solving the problem by assumptions

The data

Each Row is a Person

Y^{College} Nick	?
Y^{College} William	?
?	$Y^{\text{Non-college}}$ Rich
Y^{College} Diego	?
?	$Y^{\text{Non-college}}$ Javier
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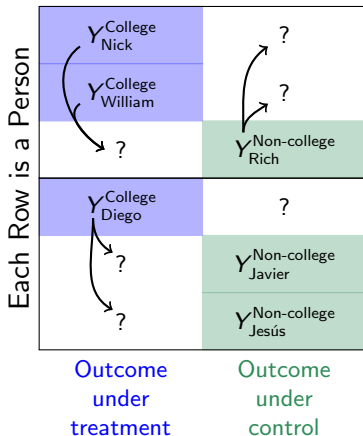
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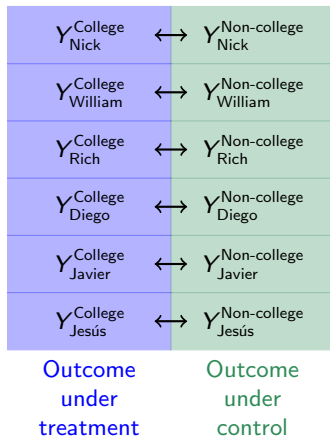
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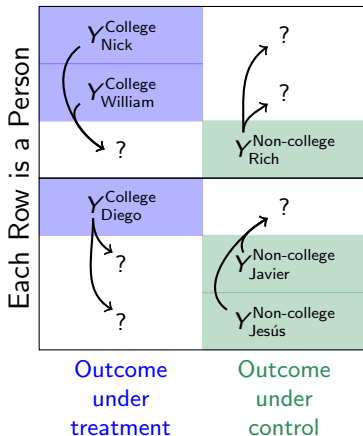


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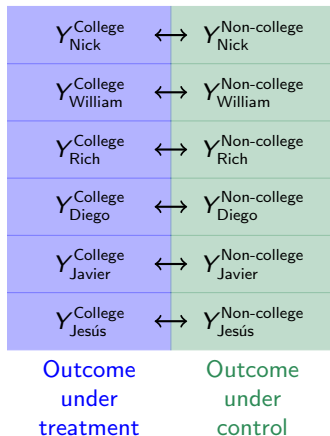


Preview: Solving the problem by assumptions

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The claim



Quick review

Quick review

1. causal effects involve missing data
 - ▶ Nick finished college college
 - ▶ outcome without college is unobserved

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 - ▶ Nick finished college college
 - ▶ outcome without college is unobserved
2. randomization solves the missing data problem by design
 - ▶ treated and control groups are exchangeable

Quick review

1. causal effects involve missing data
 - ▶ Nick finished college college
 - ▶ outcome without college is unobserved
2. randomization solves the missing data problem by design
 - ▶ treated and control groups are exchangeable
3. observational studies solve the missing data problem by assumptions
 - ▶ find population subgroups who look similar before treatment
 - ▶ assume it is like an experiment within the subgroups

Learning goals for today

By the end of class, you will be able to

- ▶ explain the fundamental problem of causal inference and the need for causal arguments
- ▶ define potential outcomes
- ▶ recall mathematical concepts from probability
 - ▶ random variables
 - ▶ expectation
 - ▶ conditional expectation

You can now

- ▶ Read Chapter 1 of [Hernán and Robins 2020](#)