Model-based causal estimation: From outcome models to treatment weighting

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Learning goals for today

At the end of class, you will be able to estimate average causal effects by modeling treatment assignment probabilities.

Optional reading:

- Hernán and Robins 2020 Chapter 12.1–12.5, 13, 15.1

Review of what we have learned

Causal assumptions

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\vec{X} \arrow[r] \arrow[rr, bend left] & A \arrow[r] & Y
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Nonparametric estimator

- Group by L, then mean difference in Y over A
- Re-aggregate over subgroups

Review of what we have learned

Causal assumptions

$$\vec{X} \longrightarrow A \longrightarrow Y$$

Nonparametric estimator

- Group by L, then mean difference in Y over A
- Re-aggregate over subgroups

Outcome modeling estimator

- Model Y^1 given L among the treated
- Model Y^0 given L among the untreated
- Predict for everyone and take the difference
- Average over all units

Inverse probability weighting: Population mean

Population Outcomes	**Randomized Sampling**	**Sampled Outcomes**
$Y_{\textbf{Maria}}$	$S_{\textbf{Maria}} = 1$	$Y_{\textbf{Maria}}$
$Y_{\textbf{William}}$	$S_{\textbf{William}} = 0$	
$Y_{\textbf{Rich}}$	$S_{\textbf{Rich}} = 0$	
$Y_{\textbf{Sarah}}$	$S_{\textbf{Sarah}} = 1$	$Y_{\textbf{Sarah}}$
$Y_{\textbf{Alondra}}$	$S_{\textbf{Alondra}} = 0$	
$Y_{\textbf{Jesús}}$	$S_{\textbf{Jesús}} = 1$	$Y_{\textbf{Jesús}}$

How many people do Maria, Sarah, and Jesús each represent?

Inverse probability weighting: Population mean

	Population Outcomes	**Randomized Sampling**	**Sampled Outcomes**
No Parent Completed College	$Y_{\textbf{Maria}}$	$S_{\textbf{Maria}} = 1$	$Y_{\textbf{Maria}}$
	$Y_{\textbf{William}}$	$S_{\textbf{William}} = 0$	
	$Y_{\textbf{Rich}}$	$S_{\textbf{Rich}} = 0$	
A Parent Completed College	$Y_{\textbf{Sarah}}$	$S_{\textbf{Sarah}} = 1$	$Y_{\textbf{Sarah}}$
	$Y_{\textbf{Alondra}}$	$S_{\textbf{Alondra}} = 0$	
	$Y_{\textbf{Jesús}}$	$S_{\textbf{Jesús}} = 1$	$Y_{\textbf{Jesús}}$

How many people do Maria, Sarah, and Jesús each represent?

Inverse probability weighting: Population mean

Each unit has a probability of being sampled

$$P(S = 1 \mid \vec{X})$$

Weight by the inverse probability of sampling

$$w = \frac{1}{\mathrm{P}(S = 1 \mid \vec{X})}$$

Inverse probability weighting: Mean under treatment

$A = 1$ indicates child completed college

Population Outcomes	**Randomized Sampling**	**Sampled Treatment**
Y^1_{Maria}	$A_{\text{Maria}} = 1$	Y^1_{Maria}
Y^1_{William}	$A_{\text{William}} = 0$	
Y^1_{Rich}	$A_{\text{Rich}} = 0$	
Y^1_{Sarah}	$A_{\text{Sarah}} = 1$	Y^1_{Sarah}
Y^1_{Alondra}	$A_{\text{Alondra}} = 0$	
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Inverse probability weighting: Mean under treatment

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	Population Outcomes	Randomized Treatment	Sampled Outcomes
No Parent Completed College	Y^1_{Maria}	$A_{\text{Maria}} = 1$	Y^1_{Maria}
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A Parent Completed College	Y^1_{Sarah}	$A_{\text{Sarah}} = 1$	Y^1_{Sarah}
	Y^1_{Alondra}	$A_{\text{Alondra}} = 0$	
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How many people do Maria, Sarah, and Jesús each represent?

Inverse probability weighting: Mean under treatment

$A = 1$ indicates child completed college. $\vec{X}$ indicates parent completed college.

When estimating the mean outcome under treatment,

$$\text{E}(Y^1)$$

each unit has a probability of being treated.

$$P(A = 1 \mid \vec{X})$$

Weight treated units by the inverse probability of treatment.

$$w = \frac{A}{\text{P}(A = 1 \mid \vec{X})}$$

Inverse probability weighting: Mean under control

$A = 1$ indicates child completed college

	Population Outcomes	Randomized Treatment	Sampled Outcomes
No Parent Completed College	Y^0_{Maria}	$A_{\text{Maria}} = 1$	
	Y^0_{William}	$A_{\text{William}} = 0$	Y^0_{William}
	Y^0_{Rich}	$A_{\text{Rich}} = 0$	Y^0_{Rich}
A Parent Completed College	Y^0_{Sarah}	$A_{\text{Sarah}} = 1$	
	Y^0_{Alondra}	$A_{\text{Alondra}} = 0$	Y^0_{Alondra}
	$Y^0_{\text{Jesús}}$	$A_{\text{Jesús}} = 1$	

How many people do William, Rich, and Alondra each represent?

Inverse probability weighting: Mean under control

$A = 1$ indicates child completed college. $\vec{X}$ indicates parent completed college.

When estimating the mean outcome under treatment,

$$\text{E}(Y^0)$$

each unit has a probability of being untreated.

$$P(A = 0 \mid \vec{X})$$

Weight treated units by the inverse probability of treatment.

$$w = \frac{1 - A}{\text{P}(A = 0 \mid \vec{X})}$$

Inverse probability weighting: Average causal effect

Define inverse probability of treatment weights

$$w_i = \begin{cases} \frac{1}{\mathrm{P}(A=1|\vec{X}=\vec{x}_i)} & \text{if treated} \\ \frac{1}{\mathrm{P}(A=0|\vec{X}=\vec{x}_i)} & \text{if untreated} \end{cases}$$

Estimate each mean potential outcome by a weighted mean

$$\hat{\mathrm{E}}(Y^1) = \sum_{i:A_i=1} w_i Y_i \quad / \quad \sum_{i:A_i=1} w_i$$

$$\hat{\mathrm{E}}(Y^0) = \sum_{i:A_i=0} w_i Y_i \quad / \quad \sum_{i:A_i=0} w_i$$

Take the difference between $\hat{\mathrm{E}}(Y^1)$ and $\hat{\mathrm{E}}(Y^0)$

What if treatment probabilities are unknown?

We need to **estimate** the probability of treatment.

Example:

- Treatment A is a college degree by age 25
- Outcome Y is spouse at age 35 has a degree
- Confounders are sex, race, mom education, dad education, income, wealth, test percentile

How would you estimate each person's probability of being treated?

Logistic regression for treatment probabilities

Model the probability of treatment

$$\underbrace{\hat{\mathrm{P}}(A = 1 \mid \vec{X})}_{\substack{\text{Estimated probability} \\ \text{of college completion}}} = \text{logit}^{-1}\left(\hat{\alpha} + \hat{\vec{\gamma}}\vec{X}\right)$$

Estimate inverse probability of treatment weights

$$\hat{w}_i = \begin{cases} \frac{1}{\hat{\mathrm{P}}(A=1 \mid \vec{X}=\vec{x}_i)} & \text{if treated} \\ \frac{1}{\hat{\mathrm{P}}(A=0 \mid \vec{X}=\vec{x}_i)} & \text{if untreated} \end{cases}$$

Estimate each mean potential outcome by a weighted mean

$$\hat{\mathrm{E}}(Y^1) = \sum_{i:A_i=1} \hat{w}_i Y_i \quad / \quad \sum_{i:A_i=1} w_i$$

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Unequal sampling and unequal treatment assignment

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Unit i was sampled with probability 0.25.

$$\mathrm{P}(S = 1 \mid \vec{X} = \vec{x}_i) = \frac{1}{4} = 0.25$$
$$w_i^{\text{Sampling}} = 4$$

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Given sampling, received treatment with probability 0.33.

$$\mathrm{P}(A = 1 \mid \vec{X} = \vec{x}_i, S = 1) = \frac{1}{3} = 0.33$$
$$w_i^{\text{Treatment}} = 3$$

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How many population Y^1 values does unit i represent?

Unequal sampling and unequal treatment assignment

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How many population Y^1 values does unit i represent?

$$w_i^{\text{Sampling}} \times w_i^{\text{Treatment}} = 4 \times 3 = 12$$

Outcome and treatment modeling: A visual summary

Outcome modeling: Model Y^0 and Y^1 given $\vec{X}$

$$\begin{tikzcd} \vec{X} \arrow[r] \arrow[rr, bend left] & A \arrow[r] & Y \end{tikzcd}$$

Treatment modeling: Model A given $\vec{X}$. Reweight.

$$\begin{tikzcd} \vec{X} \arrow[r] \arrow[rr, bend left] & A \arrow[r] & Y \end{tikzcd}$$

Original population

$$\begin{tikzcd} \vec{X} \arrow[rr, bend left] & A \arrow[r] & Y \end{tikzcd}$$

Reweighted population

What are the advantages of each strategy? How to choose?

1. Outcome modeling
   - Model Y^1 and Y^0 given $\vec{X}$
   - Predict for everyone
   - Unweighted average
2. Treatment modeling
   - Model A given X
   - Create weights: how many units each case represents
   - Weighted average

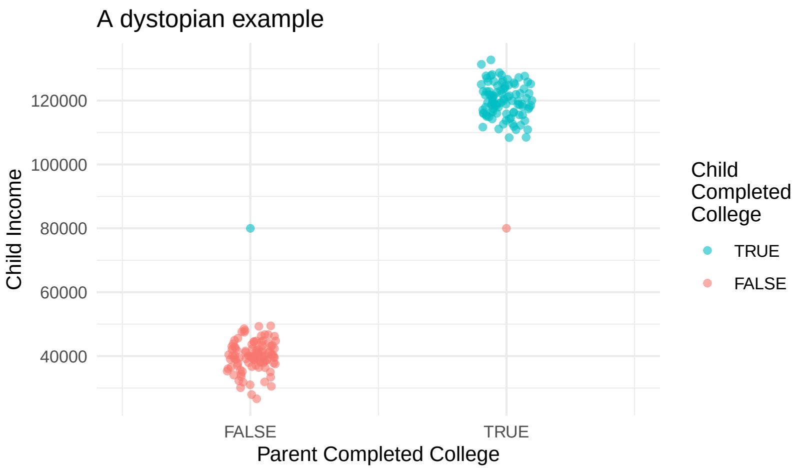
An advantage of treatment modeling

how most social scientists think about research:
model the outcome

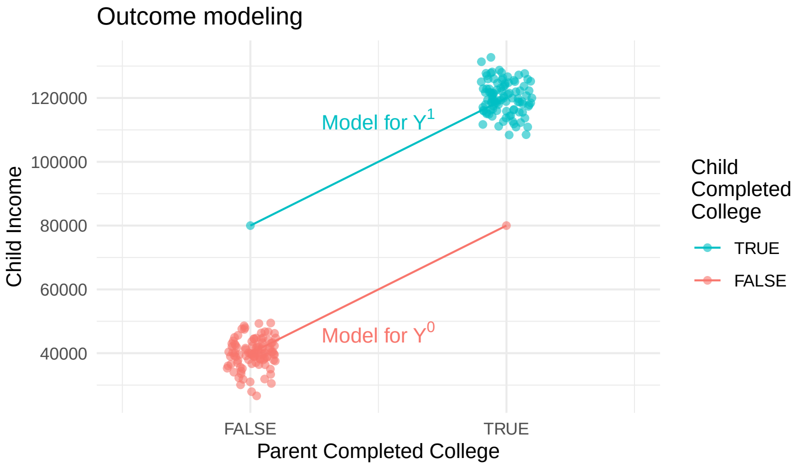
Advantages of each strategy: Treatment modeling

- how we already think about population sampling: reweight observed cases to learn about all cases
- transparency about influential observations

Transparency about influential observations



Transparency about influential observations



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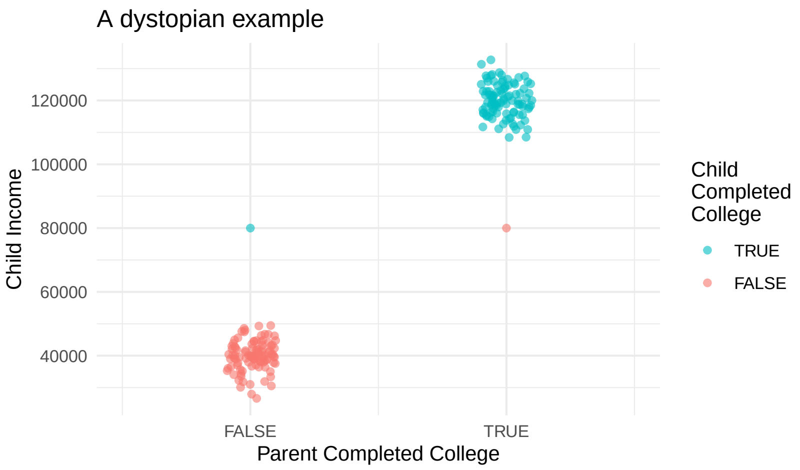

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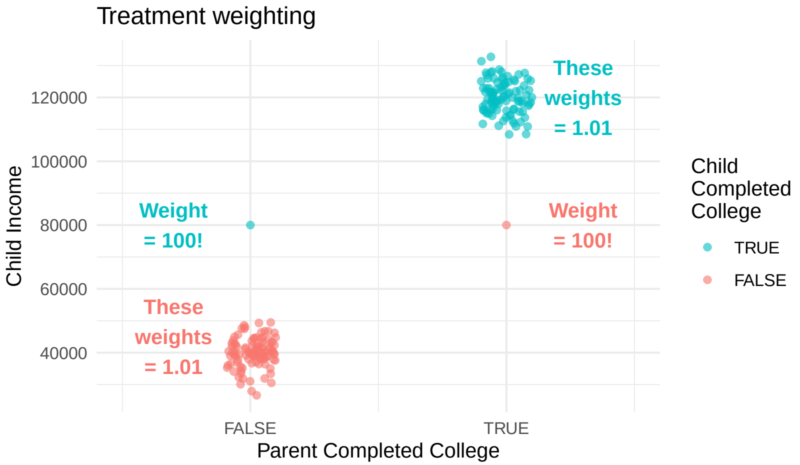
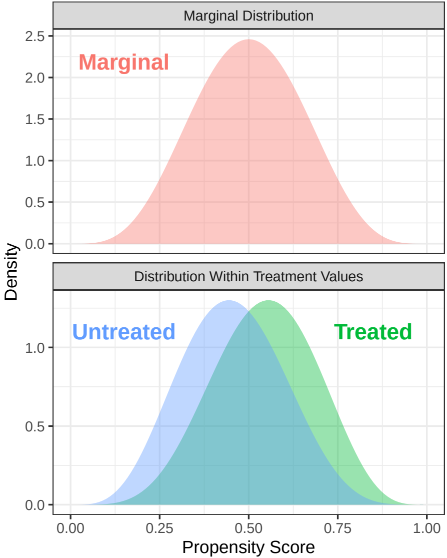


See influential observations in a real case (website).

What to do when some weights are big?

Focus on a feasible subpopulation: Region of common support



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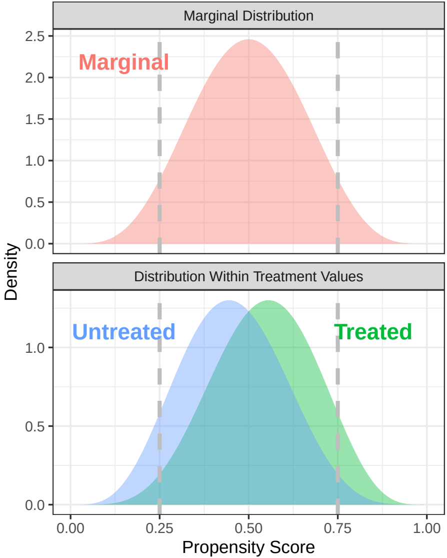

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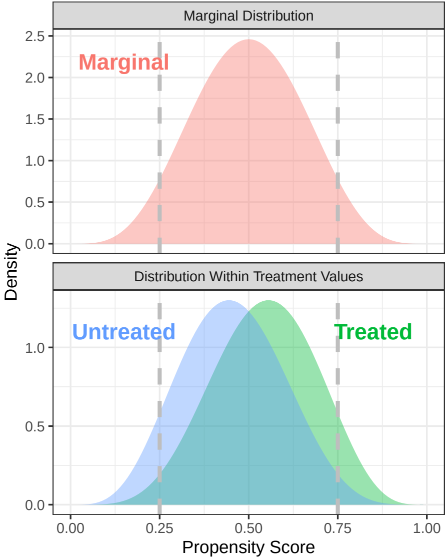

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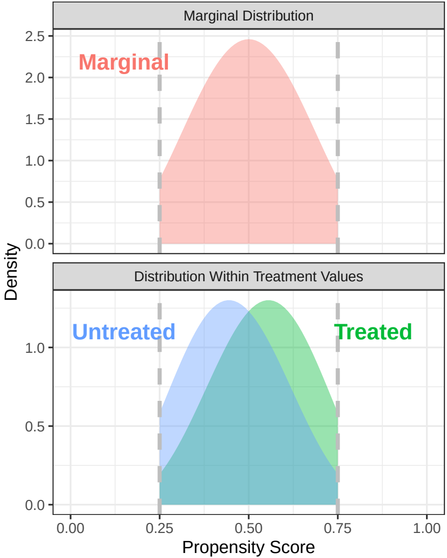

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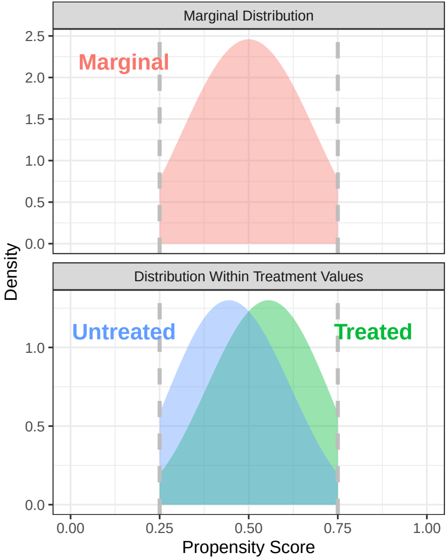

Restrict to a subgroup

Estimate in the subgroup

$$\mathrm{E}\left(Y^1 - Y^0 \mid k_1 < \mathrm{P}(A = 1 \mid \vec{X}) < k_2\right)$$

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