Conditional Exchangeability

Sociol 114

4 Feb 2025

At the end of class, you will be able to:

- 1. Define a conditionally randomized experiment
- 2. Define conditional exchangeability
- 3. Define conditional average treatment effects and recognize their use in policy

Review of exchangeability

Exchangeable sampling from a population

Population Outcomes		Randomized Sampling	Sampled Outcomes	Estin Estim
	Y _{Maria}	$S_{Maria} = 1$	Y_{Maria}	popul by the
	$Y_{William}$	$S_{\text{William}} = 0$		
	Y_{Rich}	$S_{Rich} = 0$		Key a
	Y_{Sarah}	$S_{Sarah} = 1$	Y_{Sarah}	unsan
	$Y_{Alondra}$	$S_{Alondra} = 0$		are e> due to sampl
	Y _{Jesús}	$S_{Jesús} = 1$	Y _{Jesús}	

Estimator:

Estimate the population mean by the sample mean

Key assumption: Sampled and unsampled units are exchangeable due to random sampling

 $Y \perp S$

Exchangeable treatment assignment

Population Potential Outcomes		Randomized Treatment	Obse Outc	Observed Outcomes	
Y^1_{Maria}	Y^0_{Maria}	$A_{Maria} = 1$	Y^1_{Maria}		
$Y^1_{William}$	$Y^0_{William}$	$A_{William} = 0$		Y_{William}^0	
Y^1_{Rich}	Y^0_{Rich}	$A_{Rich} = 0$		Y^0_{Rich}	
Y^1_{Sarah}	Y^0_{Sarah}	$A_{Sarah} = 1$	Y^1_{Sarah}		
$Y^1_{Alondra}$	$Y^0_{Alondra}$	$A_{\mathbf{Alondra}} = 0$		$Y^0_{Alondra}$	
$Y^1_{Jesús}$	$Y^0_{\text{Jesús}}$	$A_{ m Jesús}=1$	$Y^1_{Jesús}$		

A **conditionally** randomized experiment

A hypothetical experiment: Conditional randomization

Among the top 25% of the high school class



Among the bottom 75% of the high school class



Randomly Assigned to

High School Degree Four-Year College Degree

Outcome: Employed at age 40

Does exchangeability hold? How would you analyze?

A hypothetical experiment: Conditional randomization

Among the top 25% of the high school class



Among the bottom 75% of the high school class



Randomly Assigned to

High School Degree Four-Year College Degree

Outcome: Employed at age 40



Treated units are more likely to have done well in high school



Treated units are more likely to have done well in high school

Those who do well in high school are more likely to be employed at age 40 even without college



Treated units are more likely to have done well in high school

Those who do well in high school are more likely to be employed at age 40 even without college

$$\{Y^1, Y^0\} \not\perp A$$



Conditional randomization: Analyze within subgroups



Conditional randomization: Analyze within subgroups

Among top 25%, simple random experiment. Among bottom 75%, simple random experiment.



Conditional randomization: Analyze within subgroups

Among top 25%, simple random experiment. Among bottom 75%, simple random experiment.

Conditional exchangeability:



Conditional average treatment effects

We get two estimates. Average effect of college on employment

- ▶ among those in the top 25% of their high school class
- among those in the bottom 75% of their high school class

These are conditional average treatment effects



Effect heterogeneity: CATEs differ across subgroups

Why might the effect of college on future employment

- ▶ be larger for those from the top 25% of the high school class?
- ▶ be larger for those from the bottom 75% of the high school class?

Effect heterogeneity and policy

Suppose we study (college \rightarrow employment) in two subgroups

- Advantaged subgroup
 - Both parents finished college
 - Top quartile of family income at age 14
 - Took college prep courses
- Disadvantaged subgroup
 - Neither parent finished college
 - Bottom quartile of family income at age 14
 - Took college prep courses

Discuss:

- 1. Whose CATE would be larger?
- 2. How might the difference inform policy?

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