Defining causal effects

UCLA SOCIOL 114: Social Data Science
Winter 2026

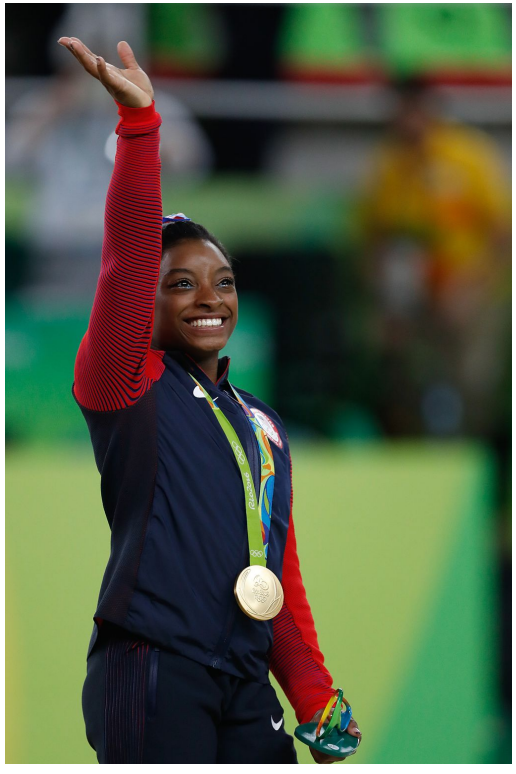
9 Feb 2026

Learning goals for today

By the end of class, you will be able to

- explain the fundamental problem of causal inference and the need for causal arguments
- define potential outcomes
- recall mathematical concepts from probability
  - random variables
  - expectation
  - conditional expectation

Causal claims hinge on arguments, not on data



Left photo: By Fernando Frazão/Agência Brasil - http://agenciabrasil.ebc.com.br/sites/_agenciabrasil2013/files/fotos/1035034-_mg_0802_04.08.16.jpg, CCBY3.0br, https://commons.wikimedia.org/w/index.php?curid=50548410
Right photo: By Agencia Brasil Fotografias - EUA levam ouro na ginástica artística feminina; Brasil fica em 8 lugar, CC BY 2.0, https://commons.wikimedia.org/w/index.php?curid=50584648

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2. Possible causal claim
   - Swinging on the uneven bars causes a person to win a gold medal.

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	Do you win gold if you:		Causal effect
	Swing	Do not swing	of swinging
Simone Biles	Yes (1)	?	?
Ian	?	No (0)	?

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Simone Biles	Yes (1)	No (0)	+1
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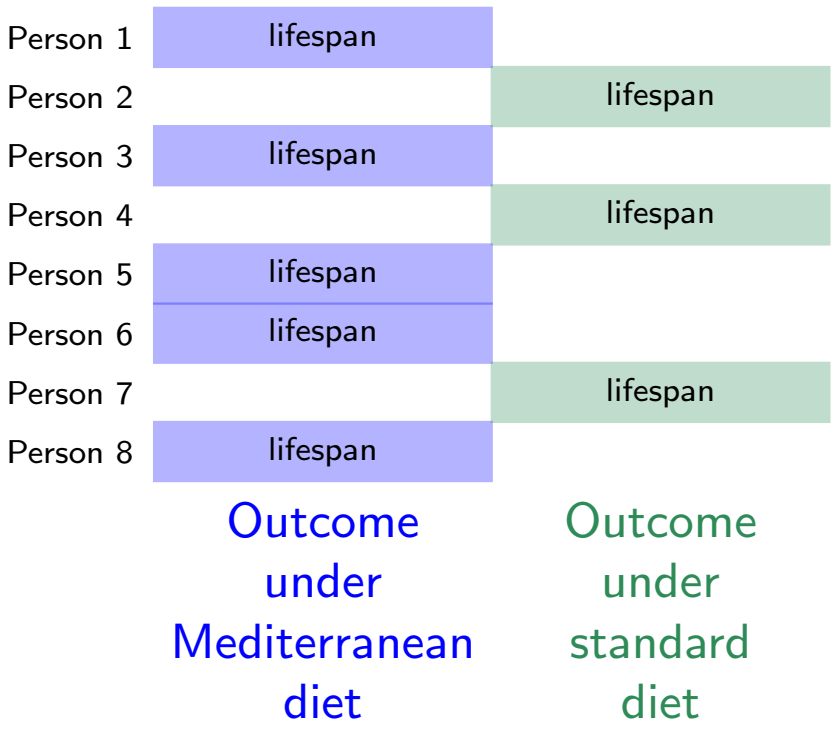
Fundamental problem of causal inference

Holland 1986

Descriptive evidence

average lifespan — average lifespan



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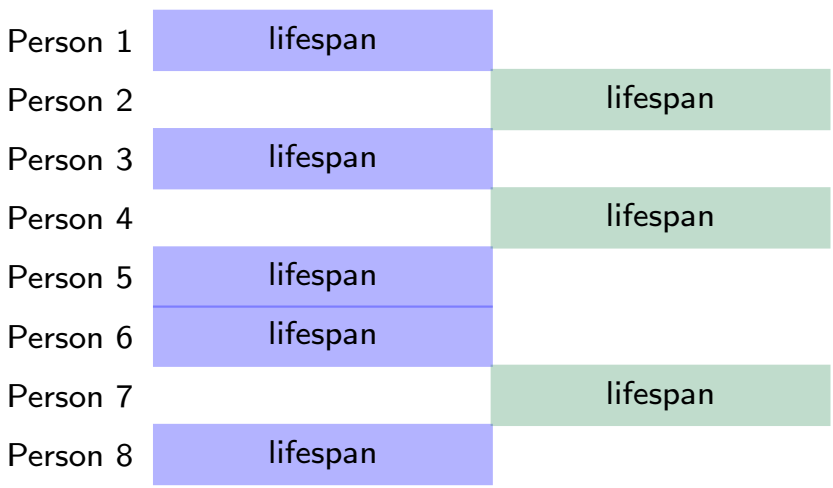

Causal claim

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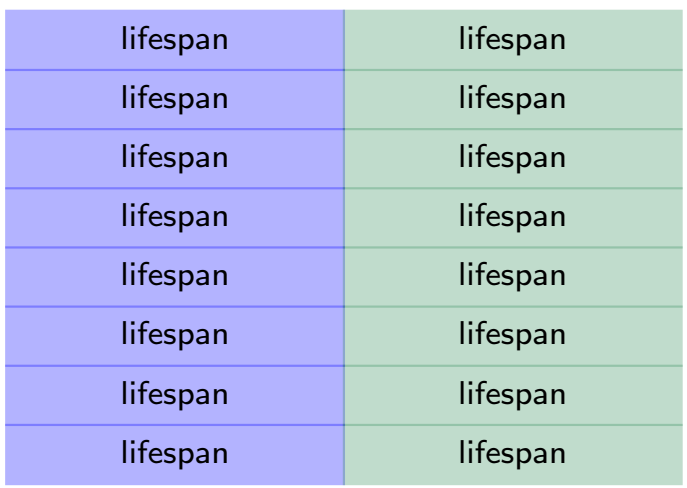

Fundamental problem of causal inference

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Descriptive evidence

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	Outcome under Mediterranean diet	Outcome under standard diet
Person 1	lifespan	missing
Person 2	missing	lifespan
Person 3	lifespan	missing
Person 4	missing	lifespan
Person 5	lifespan	missing
Person 6	lifespan	missing
Person 7	missing	lifespan
Person 8	lifespan	missing

Causal claim

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Fundamental problem of causal inference

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Causal claim

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Causal inference is a **missing data** problem

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Examples:

$Y_{\text{Ian}} = \texttt{survived}$	Ian survived

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$Y_{\text{Ian}}^{\text{MediterraneanDiet}}$	$= \texttt{survived}$	Ian would survive on a Mediterranean diet

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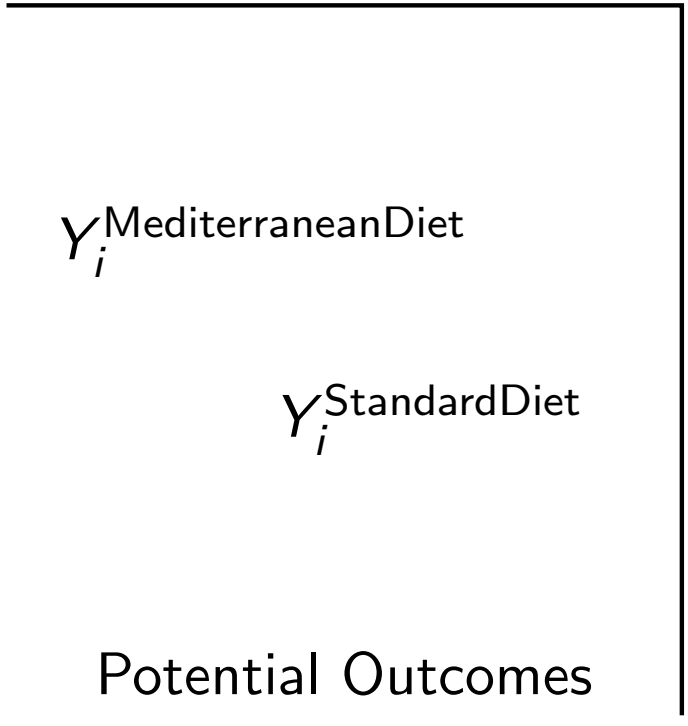
Discuss.

Which potential outcome is observed?

Which is counterfactual?

The consistency assumption

The consistency assumption



The consistency assumption

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Consistency Assumption

$$Y_i^{A_i} = Y_i$$

Mathematical notation: Potential outcomes are fixed

A person's potential outcome is a **fixed quantity**

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- Draw a random person from the population

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- The outcome $Y^{\text{MediterraneanDiet}}$ is a random variable:
  - takes the value `survived` if we randomly sample some people
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Check for understanding:

Does it make sense to write $\text{V}(Y_i^a)$? How about $\text{V}(Y^a)$

Notation: Expectation operator

The **expectation operator** E() denotes the population mean

$$\mathrm{E}(Y^a) = \frac{1}{n}\sum_{i=1}^{n} Y_i^a$$

The quantity Y^a inside the expectation must be a random variable

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A **conditional expectation** is denoted with a vertical bar

$$\mathrm{E}(Y \mid A = a) = \frac{1}{n_a}\sum_{i:A_i=a} Y_i$$

Practice: How would you say this in English?

We might wonder how a person's earnings relate to whether they hold a college degree

1. $\mathrm{E}(\text{Earnings} \mid \text{Degree} = \texttt{TRUE}) > \mathrm{E}(\text{Earnings} \mid \text{Degree} = \texttt{FALSE})$

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    - On average, a degree causes higher earnings

Practice: How would you write this in math?

1. On average, students who do the homework learn more than those who don't

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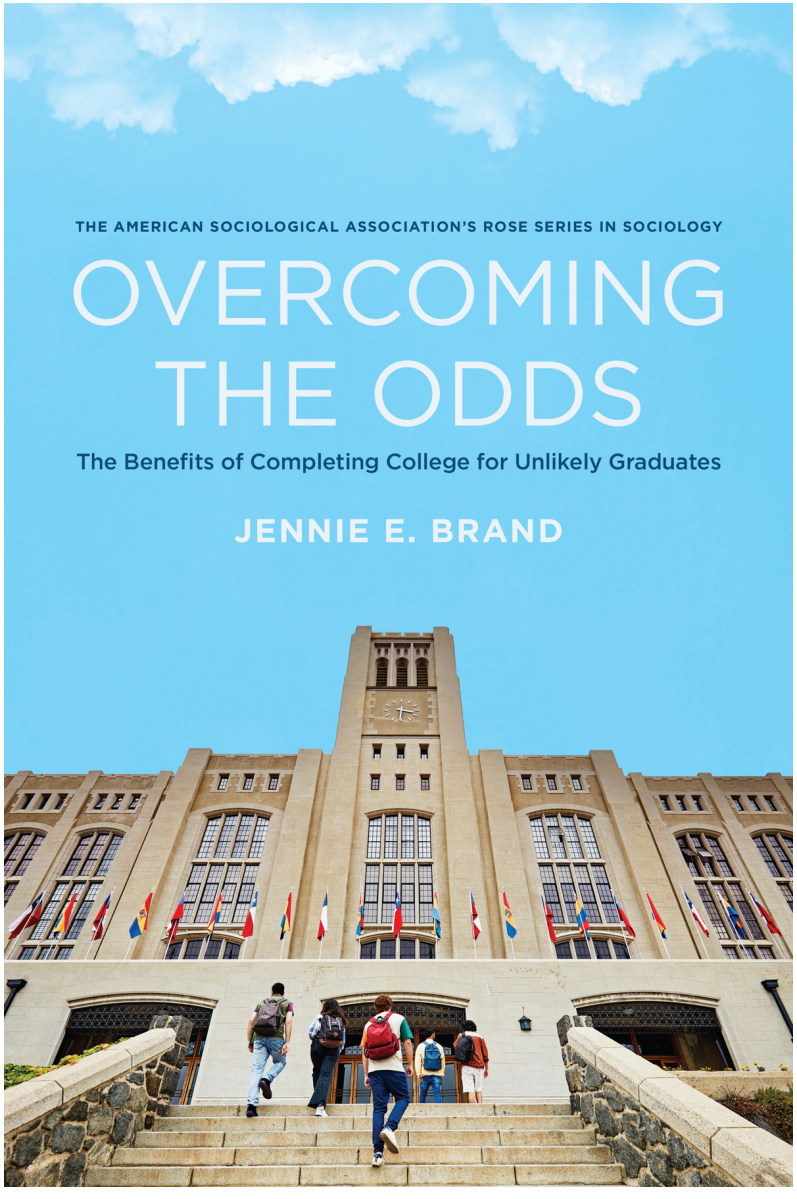
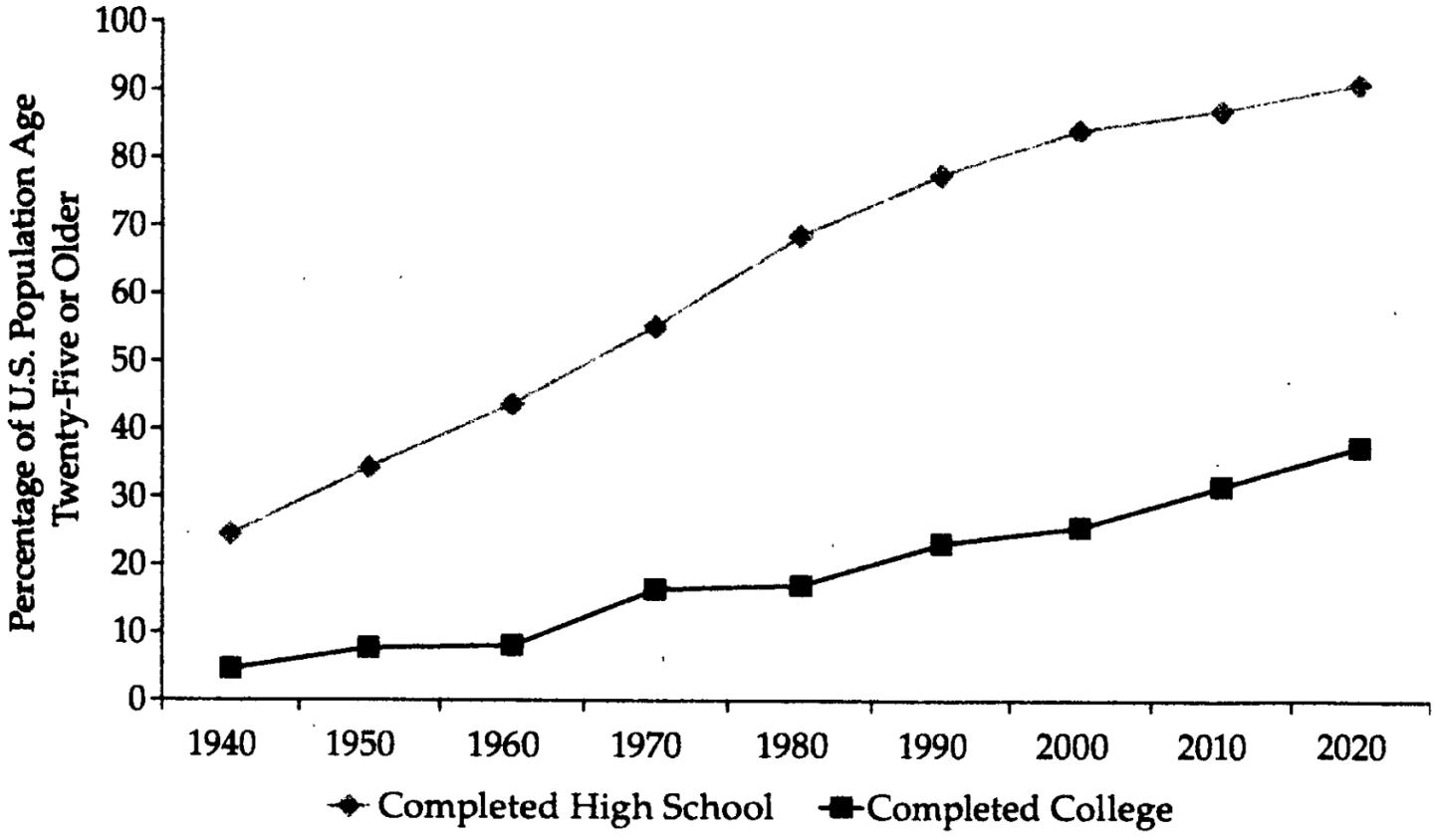
An example about inequality



Americans' education in 1900 (Brand 2023 p. 6)

- 6% graduated from high school
- 3% graduated from college

Figure 1.1 High School and Four-Year College Completion Rates, 1940–2020



Source: U.S. Census Bureau, March Current Population Survey and Annual Social and Economic Supplement to the Current Population Survey.

(Brand 2023)

We would like to know whether **college pays off**:
does it have positive effects on desired outcomes?

Mathematical notation for two types of claims

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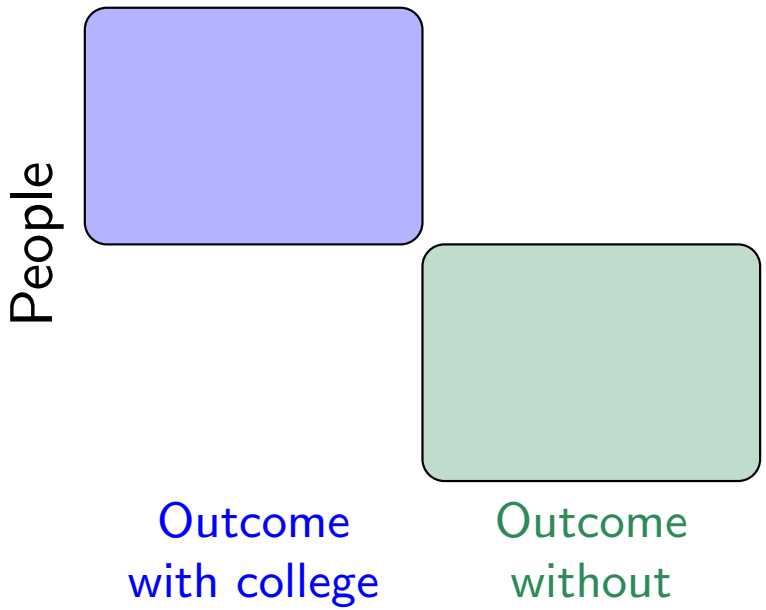

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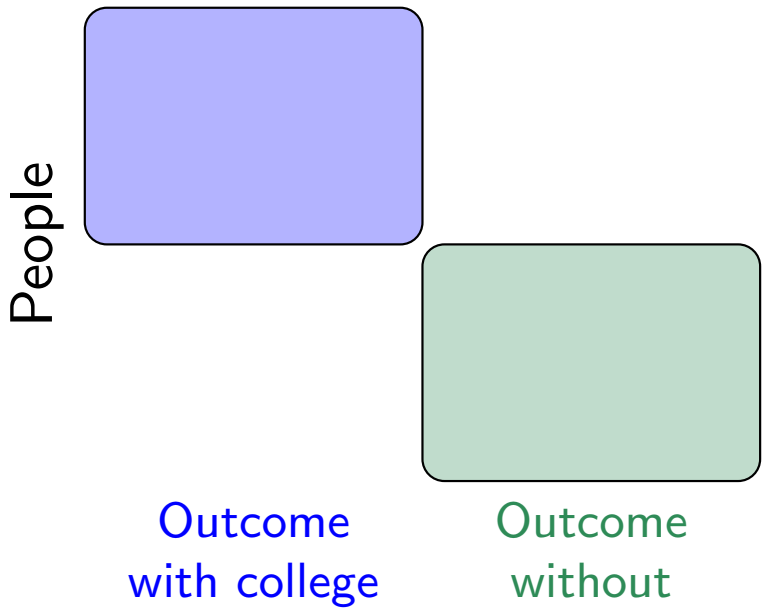

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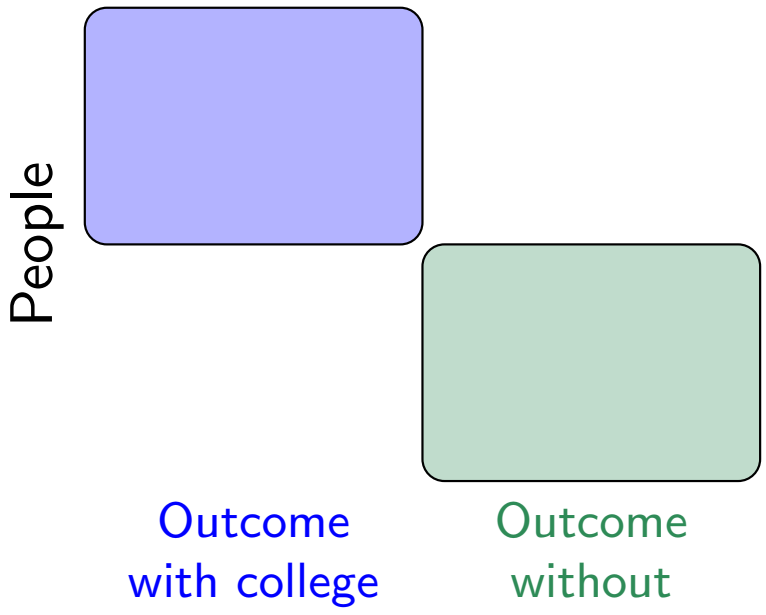

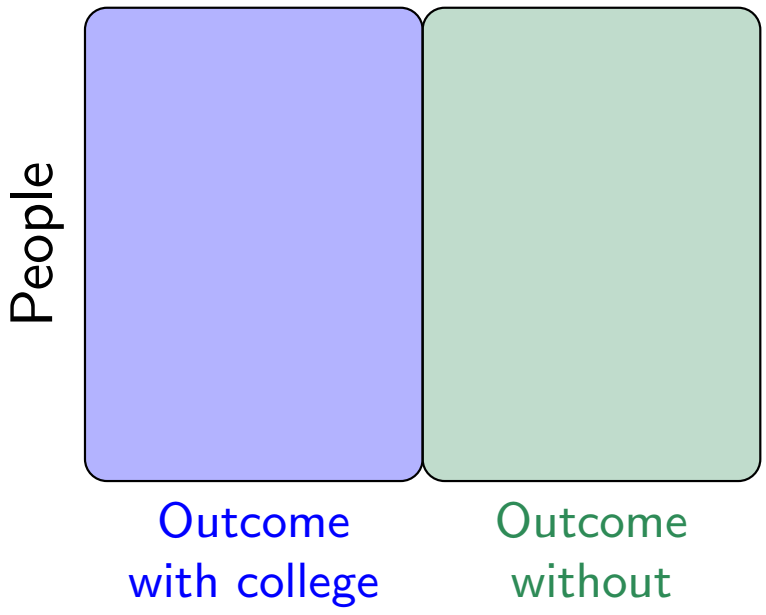

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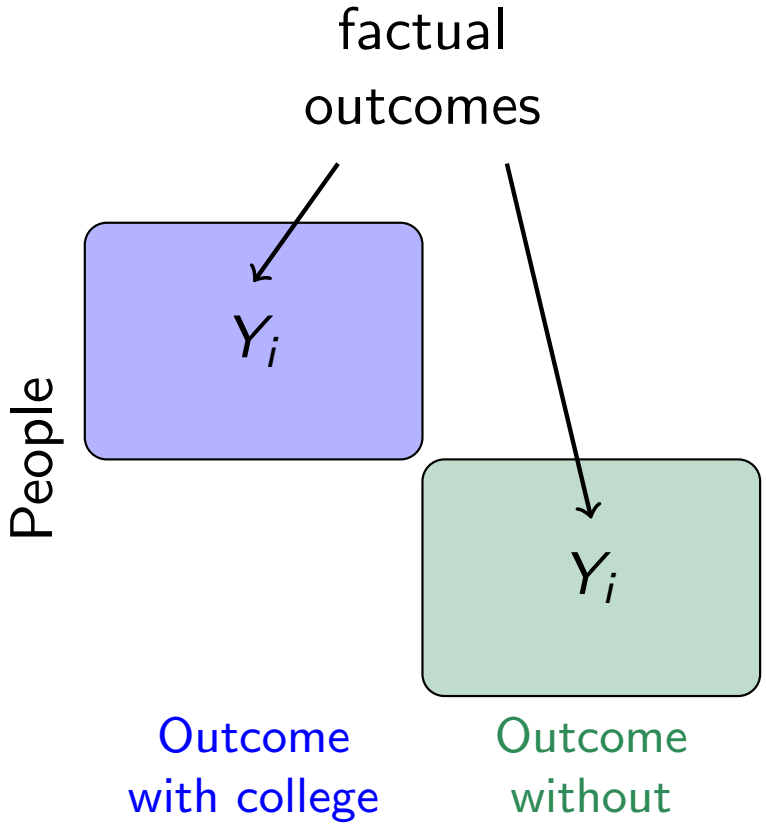

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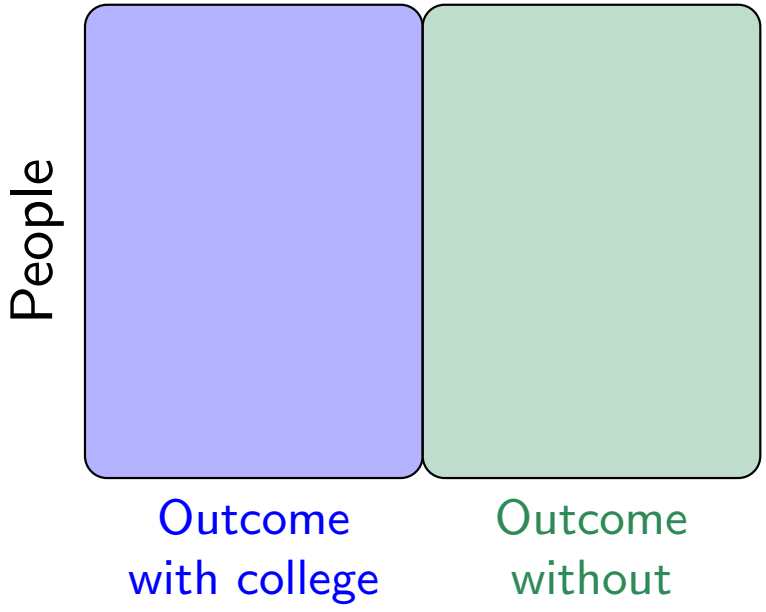

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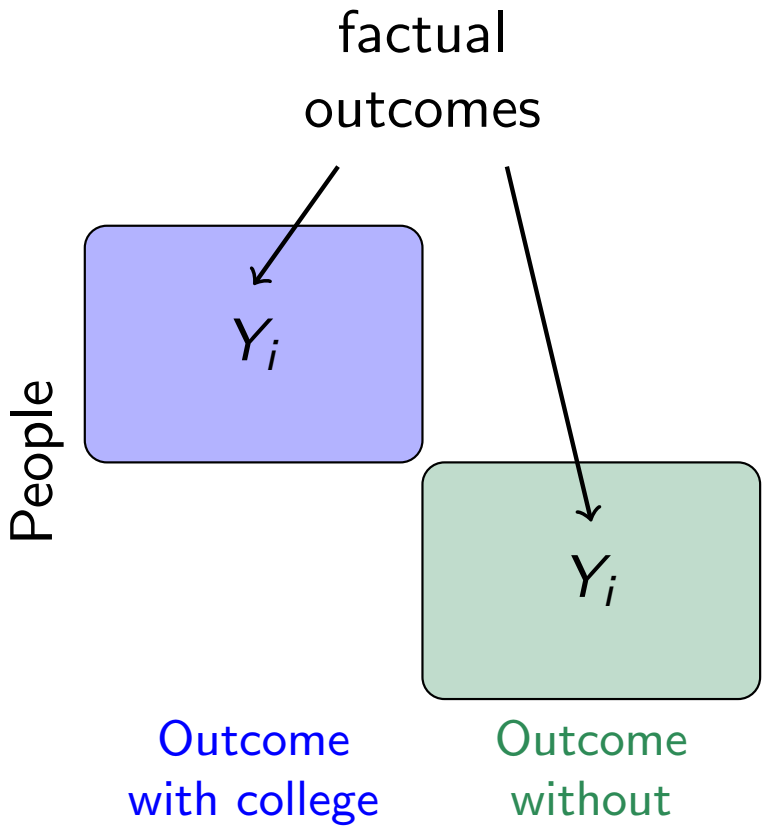
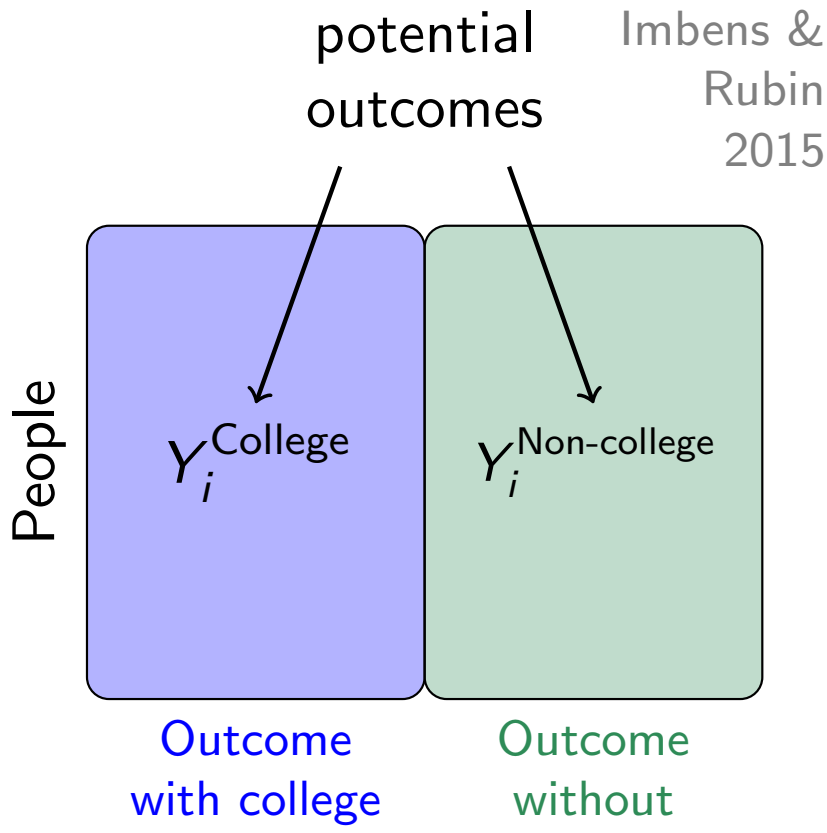
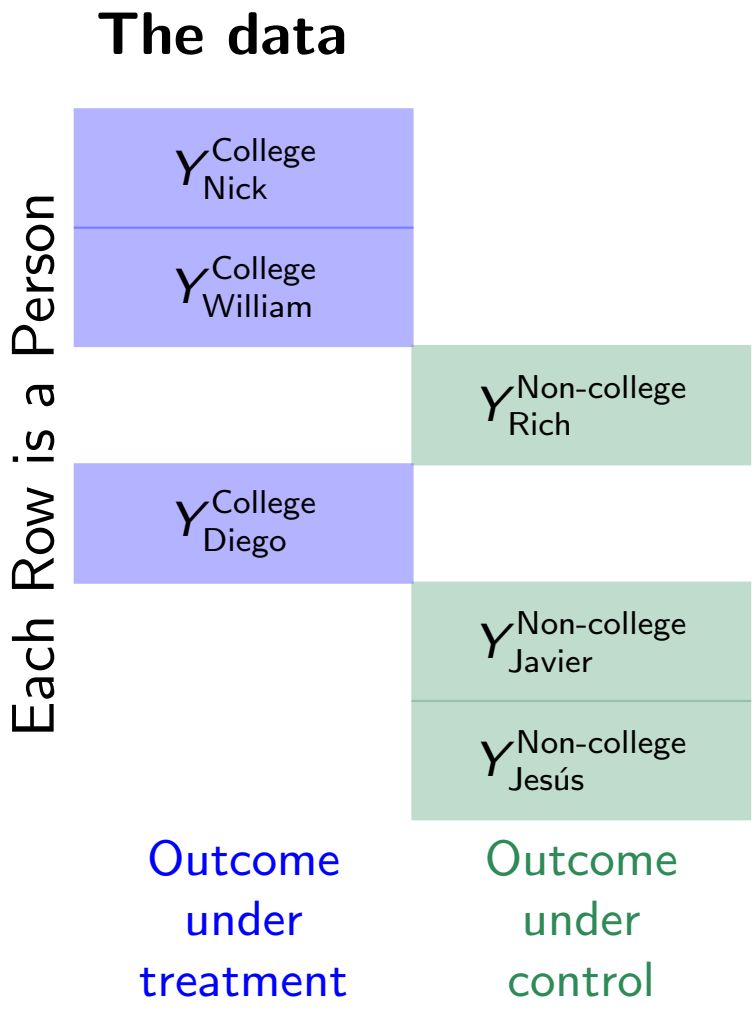




Imbens &
Rubin
2015

The fundamental problem of causal inference



Holland 1986

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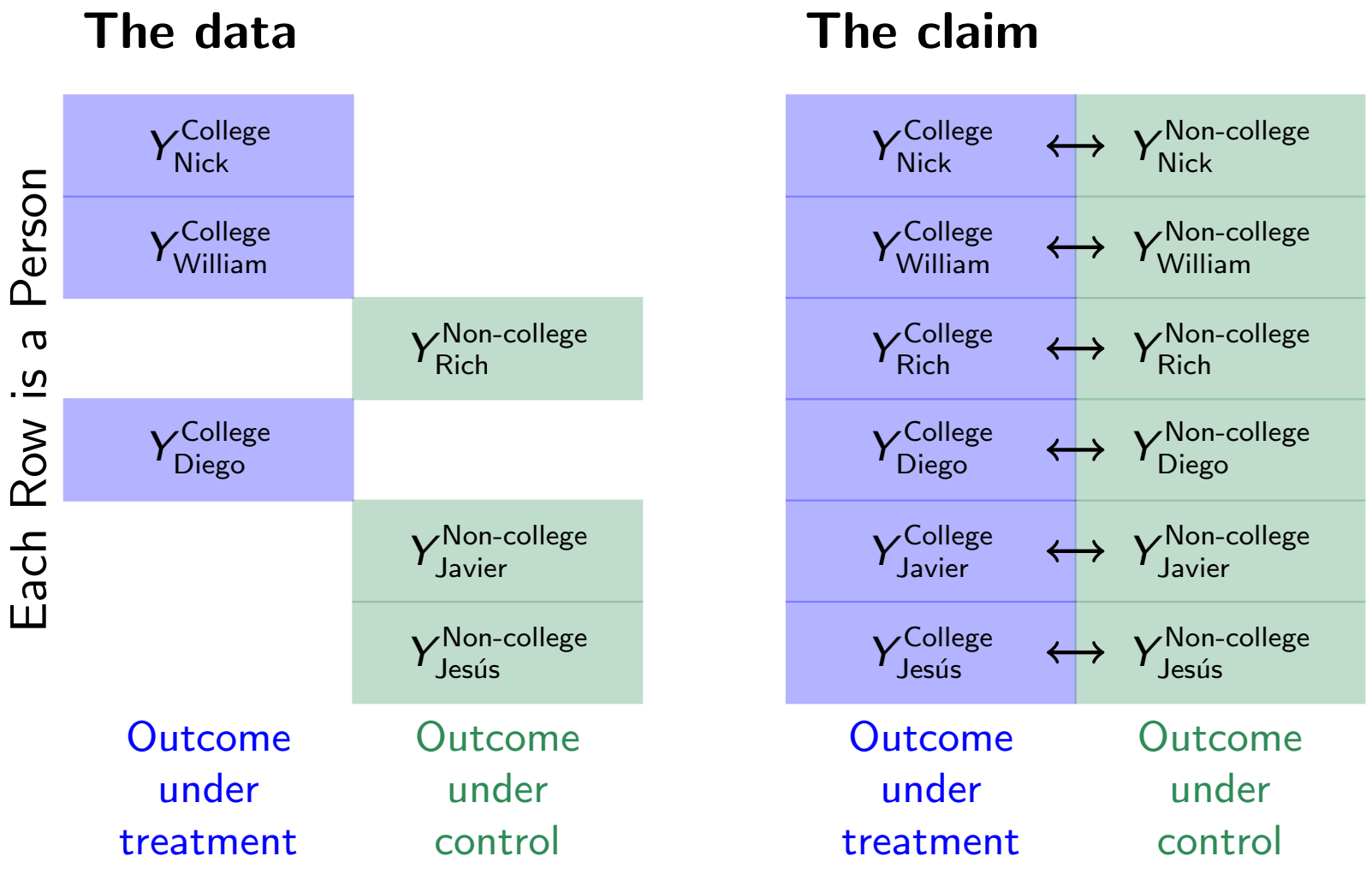

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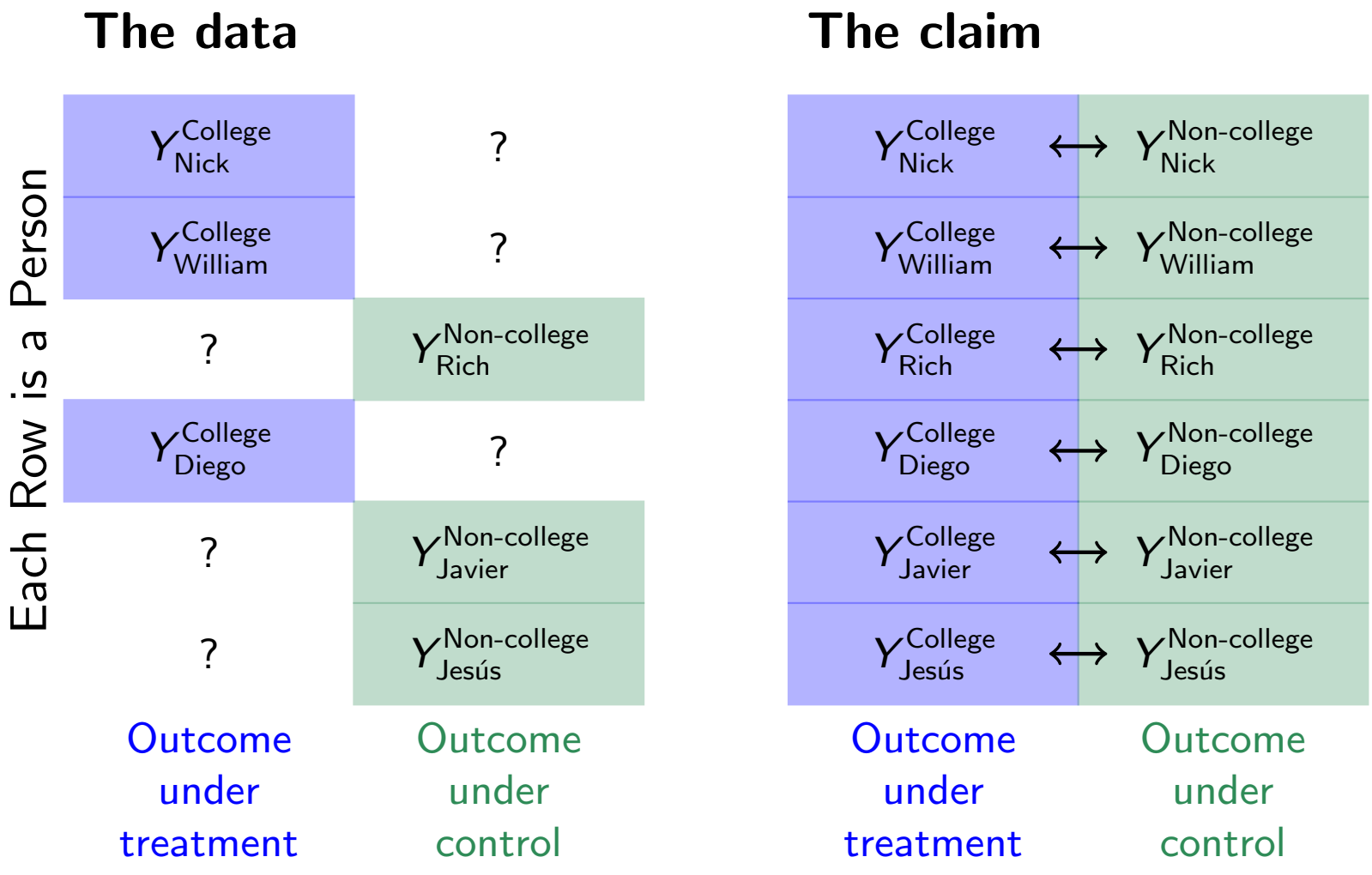
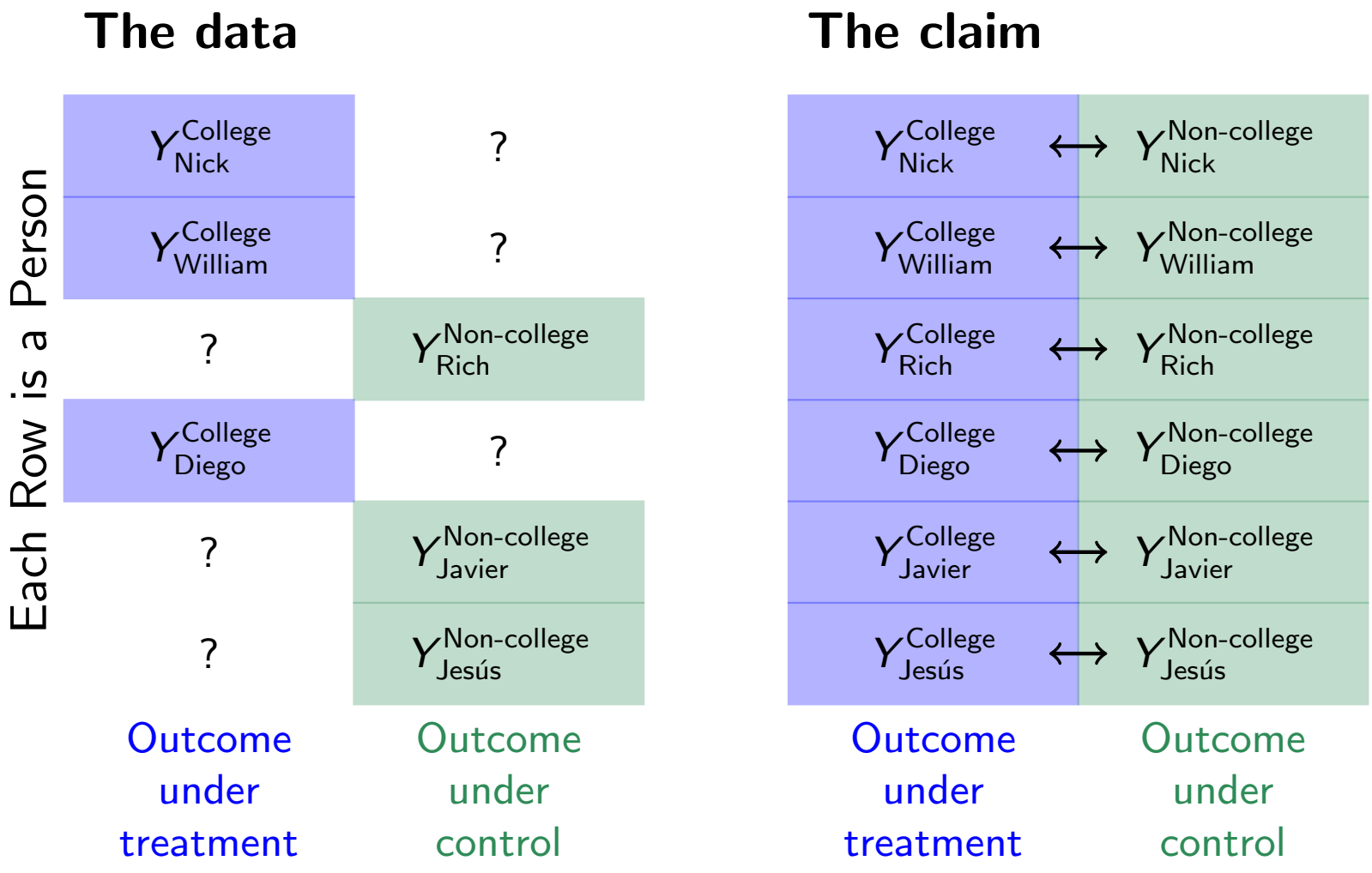


Counterfactuals are **not observed**

Holland 1986

Preview: Solving the problem by assumptions



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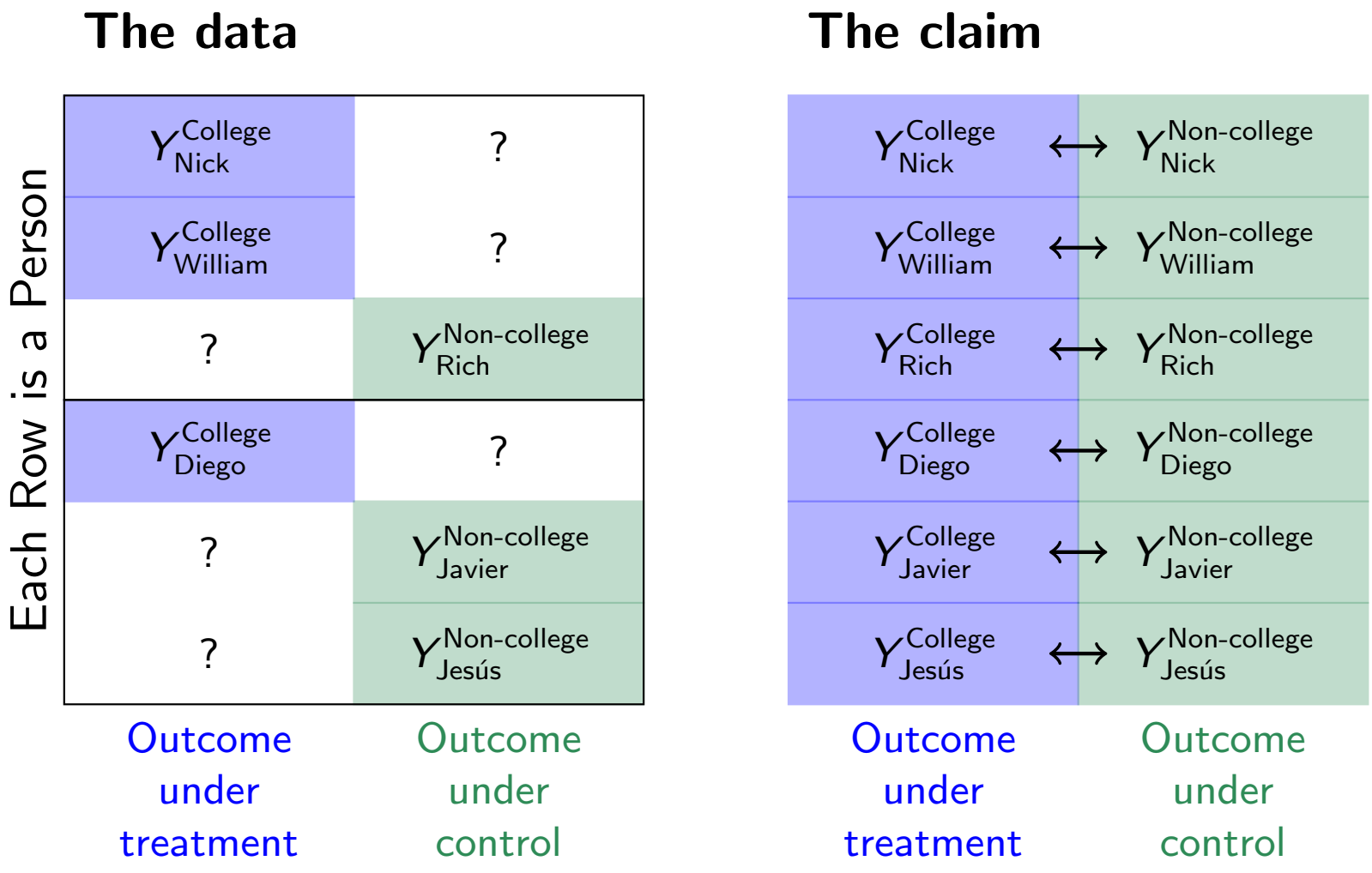

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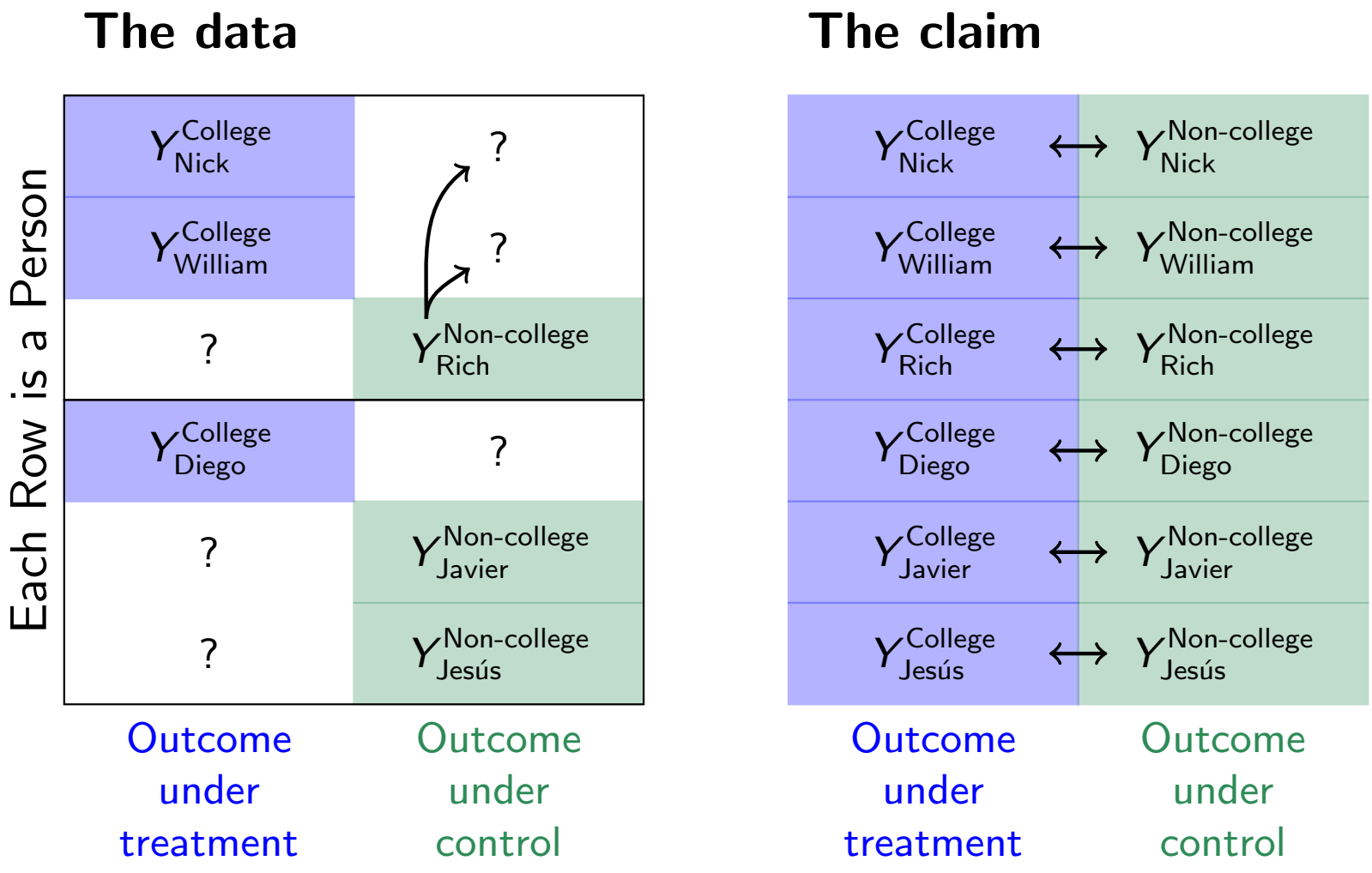

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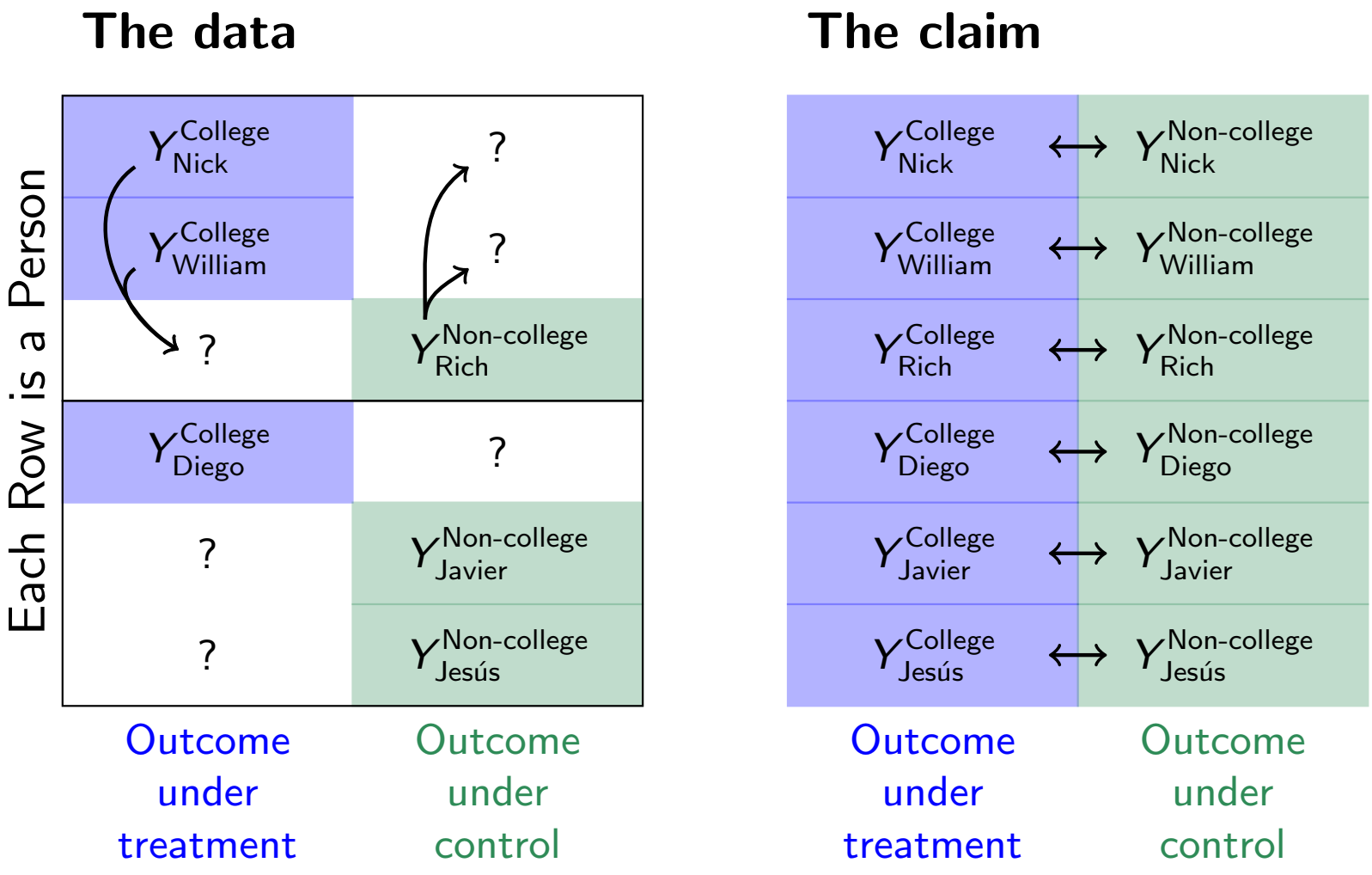

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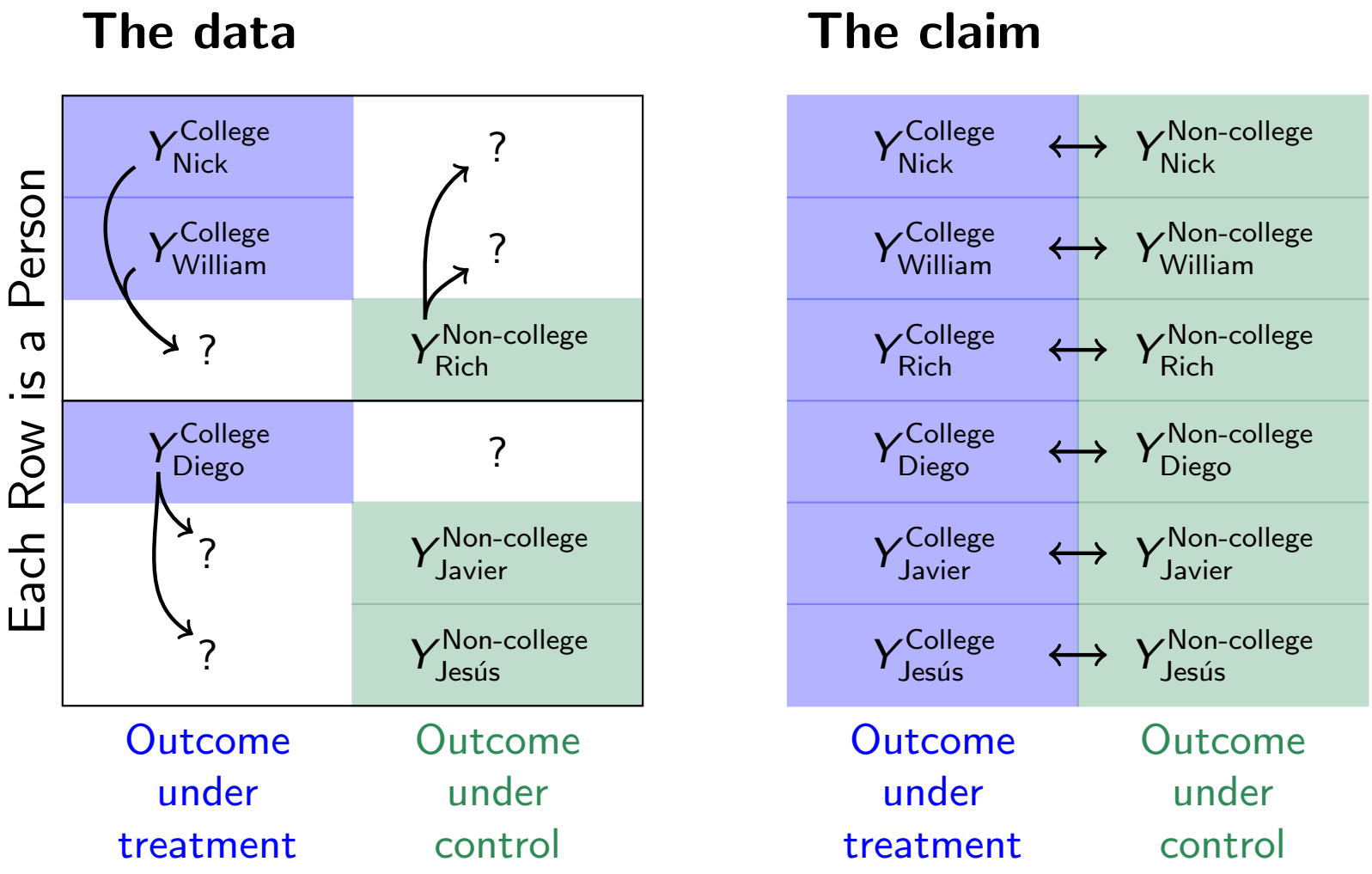

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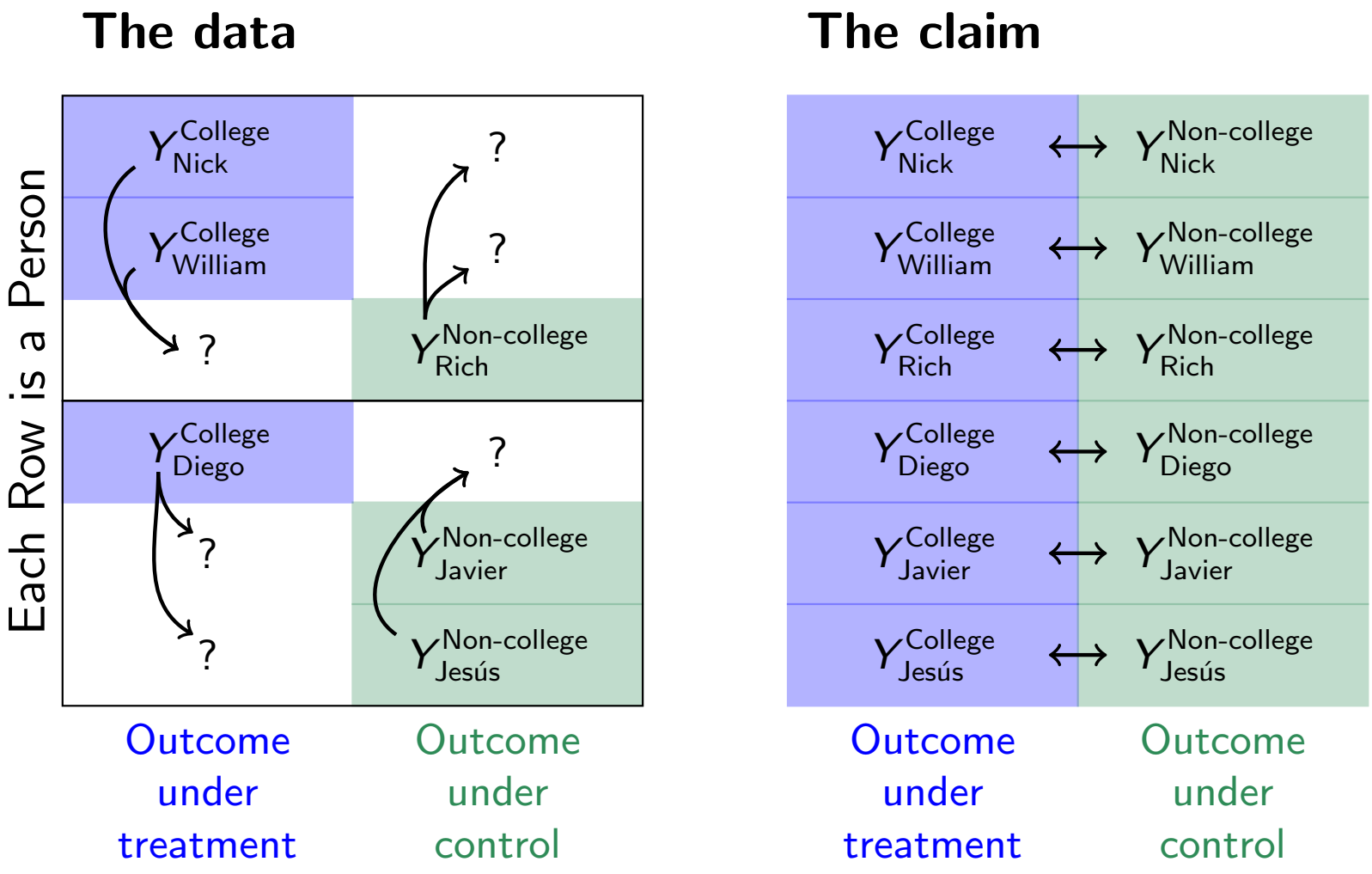

Quick review

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1. causal claims involve potential outcomes: Y^a
2. not all potential outcomes are observed
3. causal inference is a missing data problem

Learning goals for today

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- explain the fundamental problem of causal inference and the need for causal arguments
- define potential outcomes
- recall mathematical concepts from probability
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