Conditional Exchangeability

Sociol 114

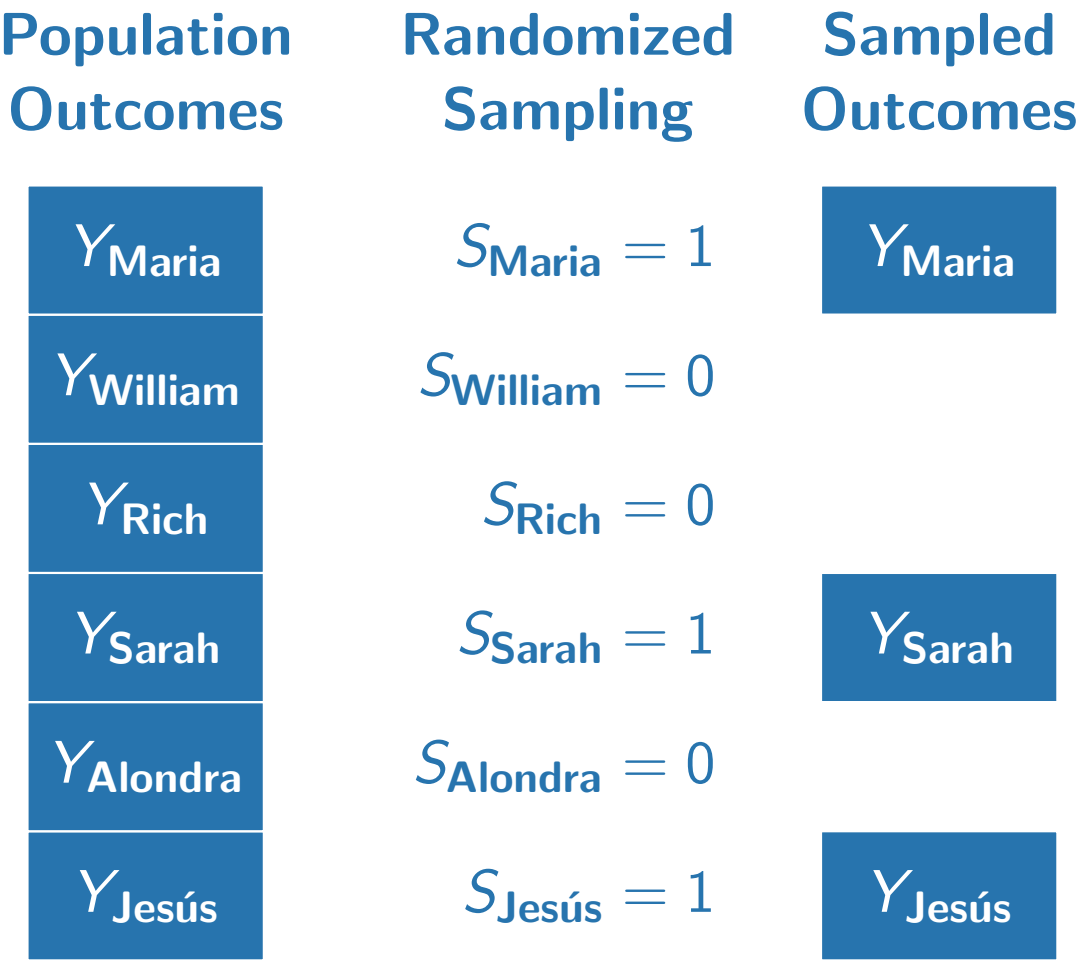
Learning goals for today

At the end of class, you will be able to:

1. Define a conditionally randomized experiment
2. Define conditional exchangeability
3. Define conditional average treatment effects and recognize their use in policy

Review of exchangeability

Exchangeable sampling from a population



Estimator:
Estimate the population mean by the sample mean

Key assumption:
Sampled and unsampled units are **exchangeable** due to random sampling

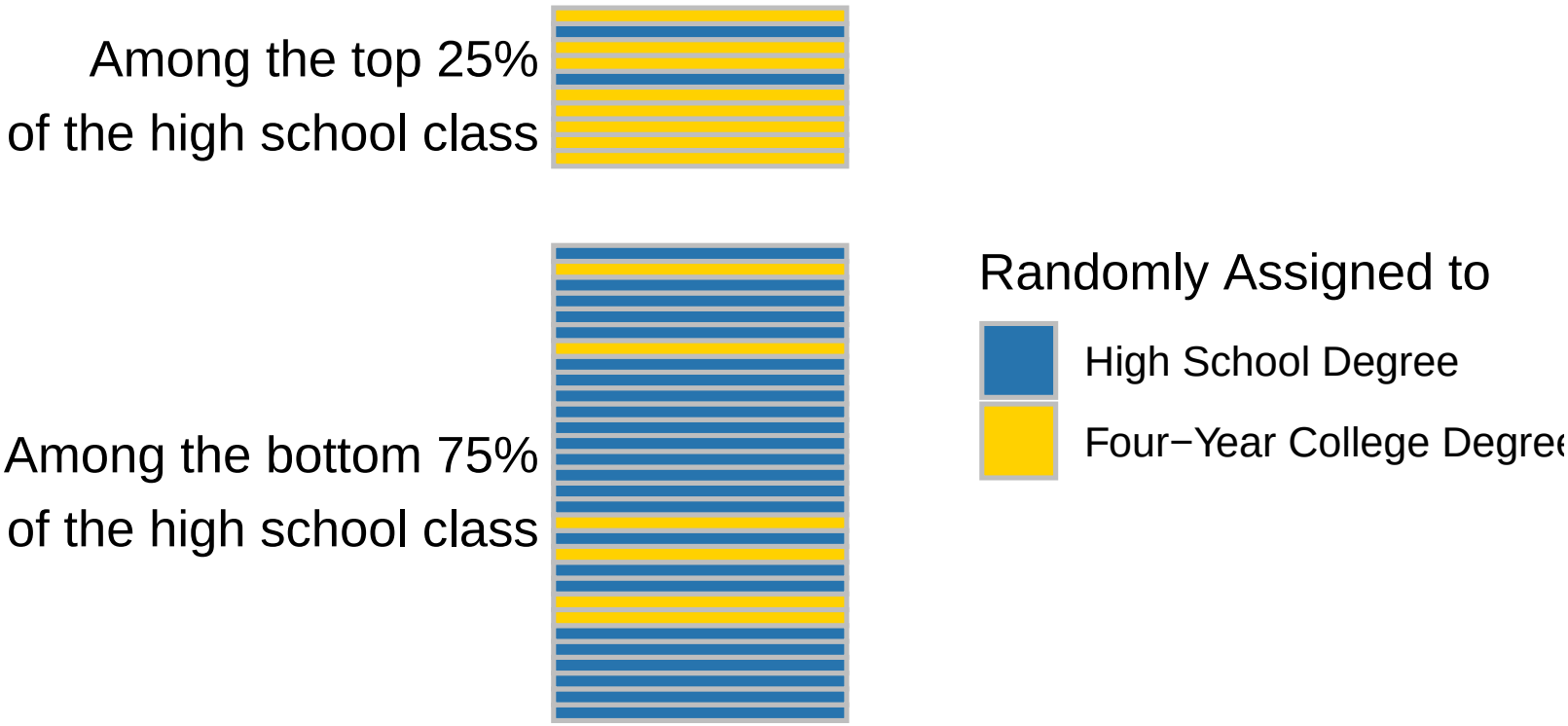
$Y \perp\!\!\!\perp S$

Exchangeable treatment assignment

Population Potential Outcomes		Randomized Treatment	Observed Outcomes	
Y^1_{Maria}	Y^0_{Maria}	$A_{\text{Maria}} = 1$	Y^1_{Maria}	
Y^1_{William}	Y^0_{William}	$A_{\text{William}} = 0$		Y^0_{William}
Y^1_{Rich}	Y^0_{Rich}	$A_{\text{Rich}} = 0$		Y^0_{Rich}
Y^1_{Sarah}	Y^0_{Sarah}	$A_{\text{Sarah}} = 1$	Y^1_{Sarah}	
Y^1_{Alondra}	Y^0_{Alondra}	$A_{\text{Alondra}} = 0$		Y^0_{Alondra}
$Y^1_{\text{Jesús}}$	$Y^0_{\text{Jesús}}$	$A_{\text{Jesús}} = 1$	$Y^1_{\text{Jesús}}$	

A **conditionally** randomized experiment

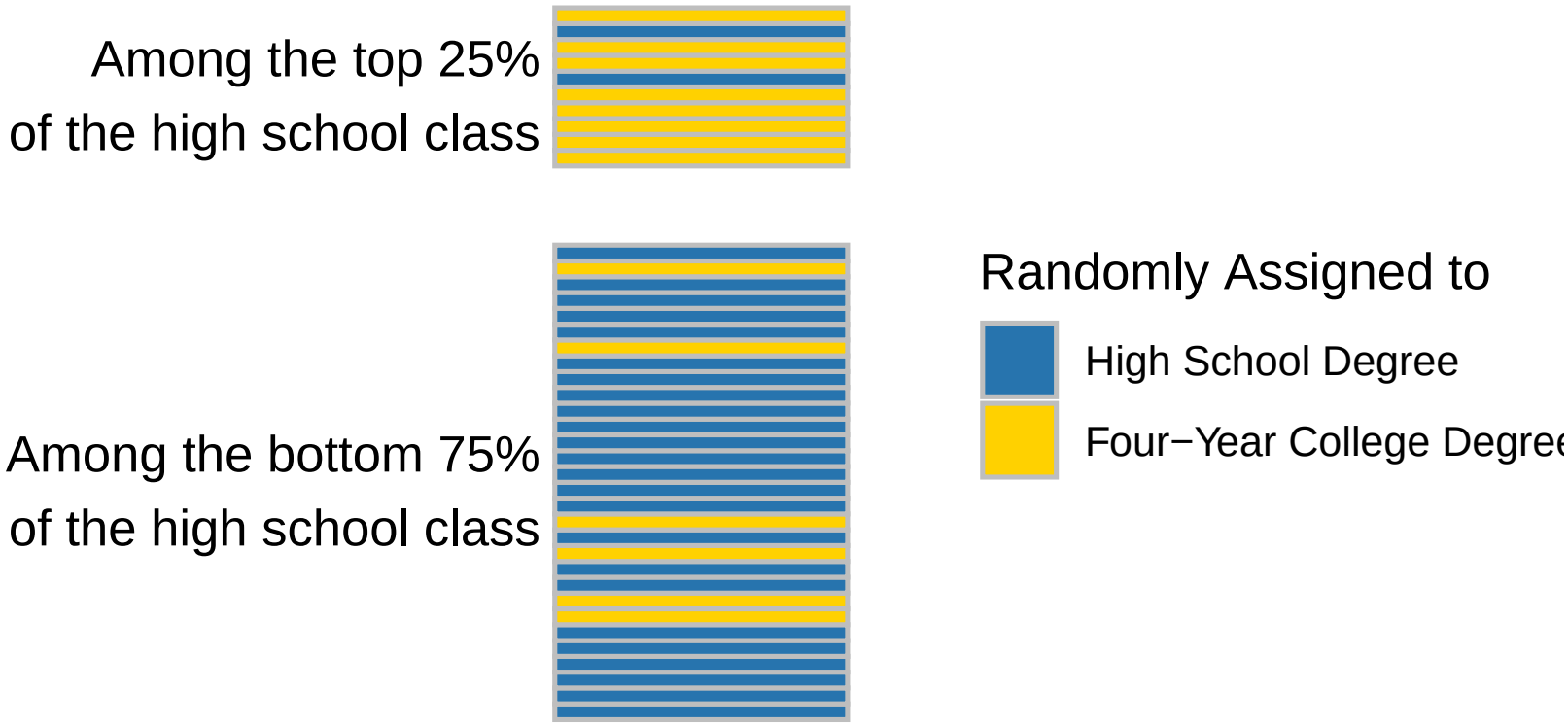
A hypothetical experiment: Conditional randomization



Outcome: Employed at age 40

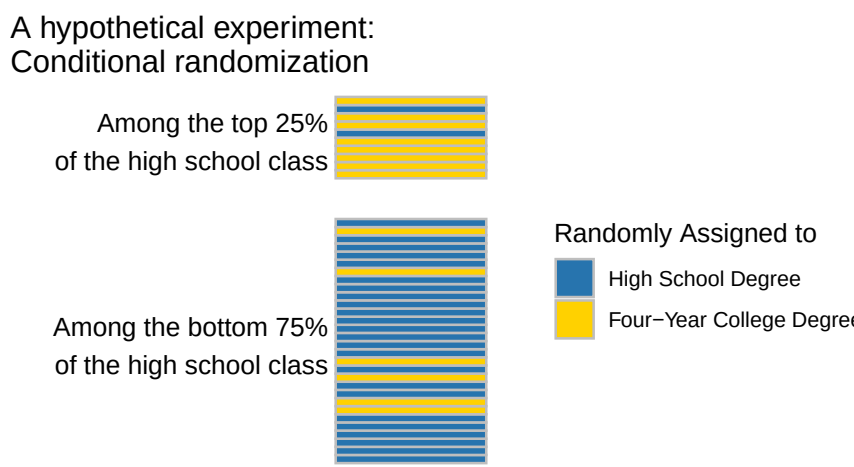
Does exchangeability hold? How would you analyze?

A hypothetical experiment:
Conditional randomization



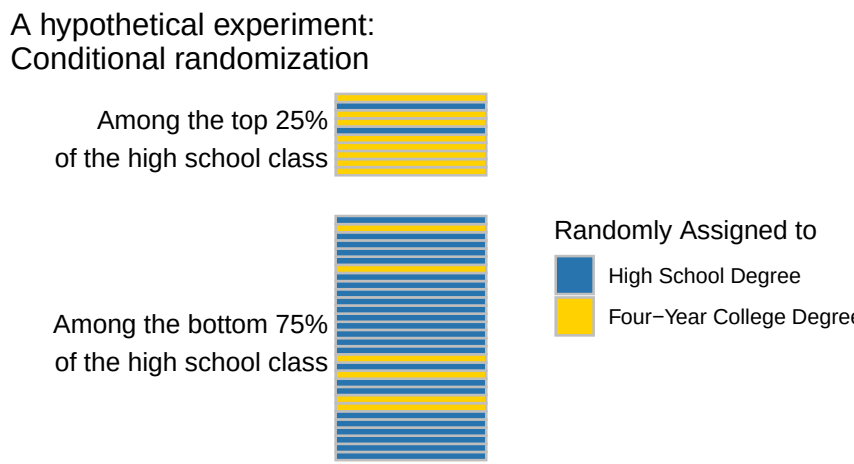
Outcome: Employed at age 40

Conditional randomization: Exchangeability does not hold



Conditional randomization: Exchangeability does not hold

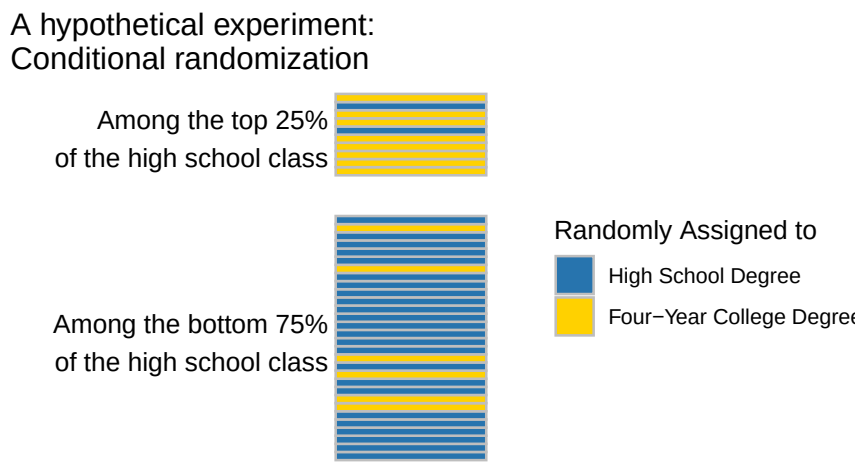
Treated units are more likely to have done well in high school



Conditional randomization: Exchangeability does not hold

Treated units are more likely to have done well in high school

Those who do well in high school are more likely to be employed at age 40 even without college

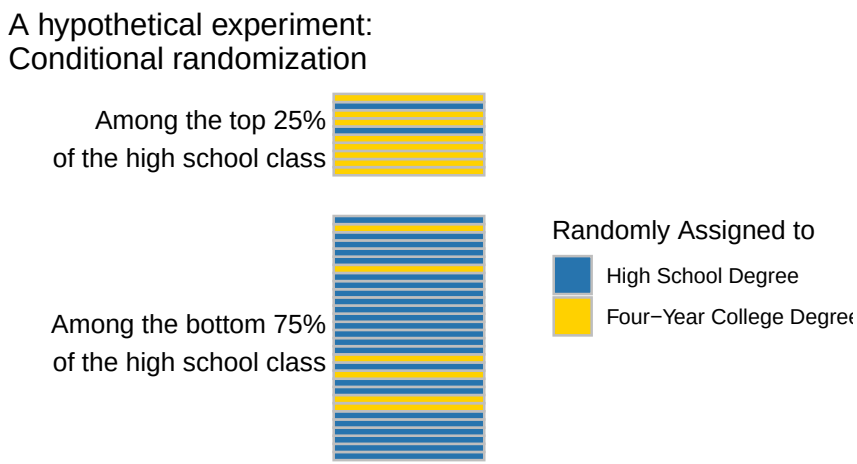


Conditional randomization: Exchangeability does not hold

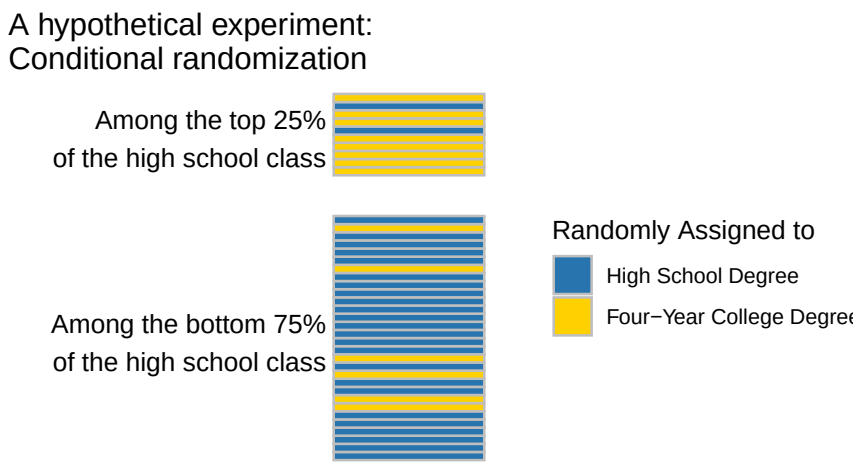
Treated units are more likely to have done well in high school

Those who do well in high school are more likely to be employed at age 40 even without college

$$\{Y^1, Y^0\} \not\perp\!\!\!\perp A$$

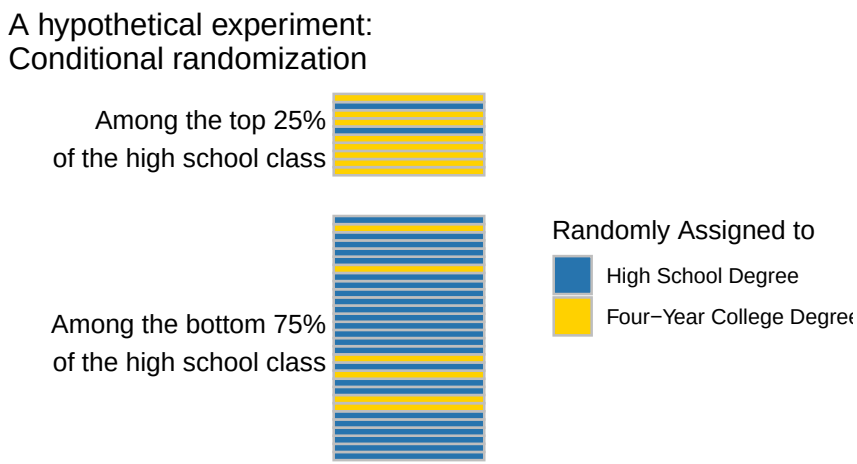


Conditional randomization: Analyze within subgroups



Conditional randomization: Analyze within subgroups

Among top 25%, simple random experiment.
Among bottom 75%, simple random experiment.

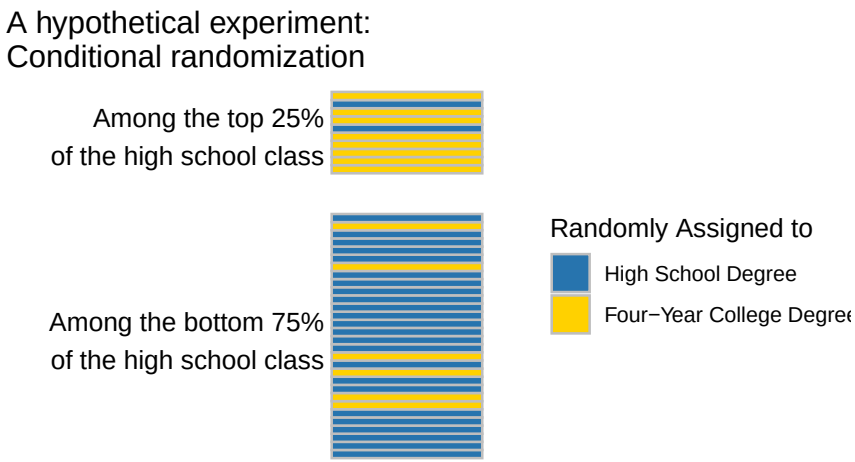


Conditional randomization: Analyze within subgroups

Among top 25%, simple random experiment.
Among bottom 75%, simple random experiment.

Conditional exchangeability:

$$\underbrace{\{Y^1, Y^0\}}_{\text{Potential Outcomes}} \underbrace{\perp\!\!\!\perp}_{\text{Are Independent of}} \underbrace{A}_{\text{Treatment}} \underbrace{|}_{\text{Within Subgroups}} \underbrace{X}_{\text{of X}}$$



Conditional average treatment effects

We get two estimates. Average effect of college on employment

- among those in the top 25% of their high school class
- among those in the bottom 75% of their high school class

These are **conditional average treatment effects**

$$\underbrace{\tau(x)}_{\substack{\text{Conditional}\\ \text{Average}\\ \text{Treatment}\\ \text{Effect}\\ \text{(CATE)}}} = \underbrace{\mathrm{E}}_{\substack{\text{Expected}\\ \text{value of}}} \left(\underbrace{Y^1 - Y^0}_{\text{treatment effect}} \quad \underbrace{|}_{\substack{\text{within the}\\ \text{subgroup}\\ \text{for whom}}} \quad \underbrace{\vec{X} = \vec{x}}_{\text{the predictors } \vec{X} \text{ take the value } \vec{x}} \right)$$

Effect heterogeneity: CATEs differ across subgroups

Why might the effect of college on future employment

- be larger for those from the top 25% of the high school class?
- be larger for those from the bottom 75% of the high school class?

Effect heterogeneity and policy

Suppose we study (college $\rightarrow$ employment) in two subgroups

- Advantaged subgroup
  - Both parents finished college
  - Top quartile of family income at age 14
  - Took college prep courses
- Disadvantaged subgroup
  - Neither parent finished college
  - Bottom quartile of family income at age 14
  - Took college prep courses

Discuss:

1. Whose CATE would be larger?
2. How might the difference inform policy?

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